

Inflation in general covariant theory of gravity

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In this paper, we study inflation in the framework of the nonrelativistic general covariant theory of the Hořava-Lifshitz gravity with the projectability condition and an arbitrary coupling constant λ . We find that the Friedmann-Robertson-Walker (FRW) universe is necessarily flat in such a setup. We work out explicitly the linear perturbations of the flat FRW universe without specifying to a particular gauge, and find that the perturbations are different from those obtained in general relativity, because of the presence of the high-order spatial derivative terms. Applied the general formulas to a single scalar field, we show that in the sub-horizon regions, the metric and scalar field are tightly coupled and have the same oscillating frequencies. In the super-horizon regions, the perturbations become adiabatic, and the comoving curvature perturbation is constant. We also calculate the power spectra and indices of both the scalar and tensor perturbations, and express them explicitly in terms of the slow roll parameters and the coupling constants of the high-order spatial derivative terms. In particular, we find that the perturbations, of both scalar and tensor, are almost scale-invariant, and, with some reasonable assumptions on the coupling coefficients, the spectrum index of the tensor perturbation is the same as that given in the minimum scenario in GR, whereas the index for scalar perturbation in general depends on λ and is different from the standard GR value. The ratio of the scalar and tensor power spectra depends on the high-order spatial derivative terms, and can be different from that of GR significantly.

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I. INTRODUCTION

The Hořava-Lifshitz (HL) theory of quantum gravity, proposed recently by Hořava [1], motivated by the Lifshitz scalar field theory in solid state physics [2], has attracted a great deal of attention, due to its several remarkable features [3, 4]. The HL theory is based on the perspective that Lorentz symmetry should appear as an emergent symmetry at long distances, but can be fundamentally absent at short ones [5]. In the latter regime, the system exhibits a strong anisotropic scaling between space and time,

$$\mathbf{x} \rightarrow \ell \mathbf{x}, \quad t \rightarrow \ell^z t, \quad (1.1)$$

where $z \geq 3$ in the $(3+1)$ -dimensional spacetime [1, 6]. At long distances, high-order curvature corrections become negligible, and the lowest order terms R and Λ take over, whereby the Lorentz invariance is expected to be “accidentally restored,” where R denotes the 3-dimensional Ricci scalar of the hypersurfaces $t = \text{Constant}$, and Λ the cosmological constant.

Because of the anisotropic scaling, the gauge symmetry of the theory is broken down to the foliation-preserving diffeomorphism, $\text{Diff}(M, \mathcal{F})$,

$$\delta t = -f(t), \quad \delta x^i = -\zeta^i(t, \mathbf{x}), \quad (1.2)$$

for which the lapse function N , shift vector N^i , and 3-spatial metric g_{ij} transform as

$$\begin{aligned} \delta N &= \zeta^k \nabla_k N + \dot{N} f + N \dot{f}, \\ \delta N_i &= N_k \nabla_i \zeta^k + \zeta^k \nabla_k N_i + g_{ik} \dot{\zeta}^k + \dot{N}_i f + N_i \dot{f}, \\ \delta g_{ij} &= \nabla_i \zeta_j + \nabla_j \zeta_i + f \dot{g}_{ij}, \end{aligned} \quad (1.3)$$

where $\dot{f} \equiv df/dt$, ∇_i denotes the covariant derivative with respect to g_{ij} , $N_i = g_{ik} N^k$, and $\delta g_{ij} \equiv \tilde{g}_{ij}(t, x^k) - g_{ij}(t, x^k)$, etc. From these expressions one can see that N and N^i play the role of gauge fields of the $\text{Diff}(M, \mathcal{F})$. Therefore, it is natural to assume that N and N^i inherit the same dependence on space and time as the corresponding generators [1],

$$N = N(t), \quad N^i = N^i(t, x), \quad (1.4)$$

which is often referred to as the projectability condition.

Due to the $\text{Diff}(M, \mathcal{F})$ diffeomorphisms (1.2), one more degree of freedom appears in the gravitational sector - a spin-0 graviton. This is potentially dangerous, and needs to decouple in the IR regime, in order to be consistent with observations. Whether this is possible or not is still an open question [3, 7]. In particular, spherically symmetric static spacetimes were studied in [4], and shown that the spin-0 graviton indeed decouples after nonlinear effects are taken into account, an analogue of the Vainshtein effect in massive gravity [8]. Along the same direction, considerations in cosmology were given in [9–11]. In particular, in [10, 11] a fully nonlinear analysis of superhorizon cosmological perturbations was carried out, by adopting the so-called gradient expansion method

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[12]. It was found that the relativistic limit is continuous, and general relativity (GR) is recovered in two different cases: (a) when only the “dark matter as an integration constant” is present [10]; and (b) when a scalar field and the “dark matter as an integration constant” are present [11].

Another very promising approach is to eliminate the spin-0 graviton by introducing two auxiliary fields, the $U(1)$ gauge field A and the Newtonian prepotential φ , by extending the $\text{Diff}(M, \mathcal{F})$ symmetry (1.2) to include a local $U(1)$ symmetry [13],

$$U(1) \ltimes \text{Diff}(M, \mathcal{F}). \quad (1.5)$$

Under this extended symmetry, the special status of time maintains, so that the anisotropic scaling (1.1) can still be realized, and the theory is kept power-counting renormalizable. Meanwhile, because of the elimination of the spin-0 graviton, its IR behavior can be significantly improved. Under the $\text{Diff}(M, \mathcal{F})$, A and φ transform as,

$$\begin{aligned} \delta A &= \zeta^i \partial_i A + \dot{f} A + f \dot{A}, \\ \delta \varphi &= f \dot{\varphi} + \zeta^i \partial_i \varphi. \end{aligned} \quad (1.6)$$

Under the local $U(1)$ symmetry, the fields transform as

$$\begin{aligned} \delta_\alpha A &= \dot{\alpha} - N^i \nabla_i \alpha, \quad \delta_\alpha \varphi = -\alpha, \\ \delta_\alpha N_i &= N \nabla_i \alpha, \quad \delta_\alpha g_{ij} = 0, \quad \delta_\alpha N = 0, \end{aligned} \quad (1.7)$$

where α is the generator of the local $U(1)$ gauge symmetry. For the detail, we refer readers to [13, 14].

The elimination of the spin-0 graviton was done initially in the case $\lambda = 1$ [13, 14], but soon generalized to the case with any λ [15–17], where λ denotes a coupling constant that characterizes the deviation of the kinetic part of action from the corresponding one given in GR with $\lambda_{GR} = 1$ (For the analysis of Hamiltonian consistency, see [13, 18]). To avoid the strong coupling problem, one may follow Blas, Pujolas and Sibiryakov (BPS) [19] to introduce an energy scale M_* that satisfies the condition [17],

$$M_* < \Lambda_\omega, \quad (1.8)$$

where M_* is the suppression energy of the sixth-order derivative terms, and Λ_ω is the would-be strong coupling energy scale, given by,

$$\Lambda_\omega \simeq \left(\frac{\zeta}{c_1} \right)^{3/2} |\lambda - 1|^{5/4} M_{\text{Pl}}, \quad (1.9)$$

where ζ is related to the Planck mass M_{Pl} through Eq.(2.6), and c_1 , defined in (B.7), represents the coupling of a scalar field with the gauge field A . In the case without the projectability condition, the observed alignment of the rotation axis of the Sun with the ecliptic requires $M_* \lesssim 10^{15}$ GeV [19]. Similar considerations have not been carried out in the current version of the HL theory, and the up bound of M_* is unknown. From the above

expression, it is clear that λ cannot be precisely equal to one, in order for the BPS mechanism to work either without [19] or with [17] the projectability condition.

It is remarkable to note that the elimination of the spin-0 graviton can be also realized in the non-projectability case with the extended symmetry (1.5) [20, 21]. In addition, the number of independent coupling constants can be significantly reduced (from more than 70 [22] to 15), by simply imposing the softly breaking detailed balance condition, while the theory still remains power-counting renormalizable and has a healthy IR limit.

In this paper, we study inflation of a scalar field in the Hořava and Melby-Thompson (HMT) setup with the projectability condition [13] and an arbitrary coupling constant λ [15]. Specifically, after a brief review of the theory in Sec. II, we first show that the FRW universe is necessarily flat, when it is filled with (multi-) scalar, vector or fermionic fields in Sec. III.A. Then, in the second part of Sec. III we present the general linear scalar perturbations without specifying to a particular gauge or specific matter fields, while in the third part of it, we consider several possible gauge choices. Unlike the case without the $U(1)$ symmetry [23], some gauges used in GR [24], such as the longitudinal gauge, now become possible, because of the $U(1)$ gauge freedom. In Sec. IV, we first consider the flat FRW background, and show clearly that the slow-roll conditions imposed in GR are also needed here, in order to obtain enough e-fold to solve the problems such as horizon, monopole, domain walls, and so on [25]. In addition, in this section we also show that in the super-horizon regions, the perturbations become adiabatic, and the comoving curvature perturbation is constant. In Sec. V, we show explicitly that in the sub-horizon regions, the metric and scalar field are tightly coupled and have the same oscillating frequencies, while in the super-horizon regions, the perturbations are almost scale-invariant. It is remarkable that a master equation for the scalar perturbations exists, in contrast to the case without the $U(1)$ symmetry [23]. In Sec. VI, we calculate the power spectra and indices of both scalar and tensor perturbations in the slow-roll approximations, by using the uniform approximation [26]. We express them explicitly in terms of the slow roll parameters and the coupling constants of the high-order spatial derivative terms. Finally, in Sec. VII we present our main conclusions.

II. GENERAL COVARIANT THEORY WITH AN ARBITRARY CONSTANT λ

In this section, we shall give a very brief introduction to the HMT setup with the projectability condition and $\lambda \neq 1$. For detail, we refer readers to [13, 15, 16].

The total action of the theory can be written as,

$$S = \zeta^2 \int dt d^3x N \sqrt{g} \left(\mathcal{L}_K - \mathcal{L}_V + \mathcal{L}_\varphi + \mathcal{L}_A + \mathcal{L}_\lambda \right)$$

$$+\zeta^{-2}\mathcal{L}_M), \quad (2.1)$$

where $g = \det(g_{ij})$, and

$$\begin{aligned} \mathcal{L}_K &= K_{ij}K^{ij} - \lambda K^2, \\ \mathcal{L}_\varphi &= \varphi \mathcal{G}^{ij} \left(2K_{ij} + \nabla_i \nabla_j \varphi \right), \\ \mathcal{L}_A &= \frac{A}{N} (2\Lambda_g - R), \\ \mathcal{L}_\lambda &= (1 - \lambda) \left[(\Delta\varphi)^2 + 2K\Delta\varphi \right]. \end{aligned} \quad (2.2)$$

Here $\Delta \equiv g^{ij} \nabla_i \nabla_j$, Λ_g is a coupling constant, the Ricci and Riemann tensors R_{ij} and R^i_{jkl} all refer to the 3-metric g_{ij} , and

$$\begin{aligned} K_{ij} &= \frac{1}{2N} (-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i), \\ \mathcal{G}_{ij} &= R_{ij} - \frac{1}{2} g_{ij} R + \Lambda_g g_{ij}. \end{aligned} \quad (2.3)$$

\mathcal{L}_M is the Lagrangian of matter fields, which is a scalar not only with respect to the $\text{Diff}(M, \mathcal{F})$ symmetry (1.2), but also to the $U(1)$ symmetry (1.7). \mathcal{L}_V is an arbitrary $\text{Diff}(\Sigma)$ -invariant local scalar functional built out of the spatial metric, its Riemann tensor and spatial covariant derivatives, without the use of time derivatives. Assuming that the highest order derivatives are six, and that the theory respects the parity and time-reflection symmetry the most general form of \mathcal{L}_V is given by [27, 28],

$$\begin{aligned} \mathcal{L}_V &= \zeta^2 g_0 + g_1 R + \frac{1}{\zeta^2} (g_2 R^2 + g_3 R_{ij} R^{ij}) \\ &+ \frac{1}{\zeta^4} \left(g_4 R^3 + g_5 R R_{ij} R^{ij} + g_6 R^i_j R^j_k R^k_i \right) \\ &+ \frac{1}{\zeta^4} [g_7 R \Delta R + g_8 (\nabla_i R_{jk}) (\nabla^i R^{jk})], \end{aligned} \quad (2.4)$$

where the coupling constants g_s ($s = 0, 1, 2, \dots, 8$) are all dimensionless, and

$$\Lambda = \frac{1}{2} \zeta^2 g_0, \quad (2.5)$$

is the cosmological constant. The relativistic limit in the IR, on the other hand, requires,

$$g_1 = -1, \quad \zeta^2 = \frac{1}{16\pi G} = \frac{M_{\text{pl}}^2}{2}, \quad (2.6)$$

where G is the Newtonian constant.

Note the difference between the notations used here and the ones used in [13, 15]¹. In this paper, we shall use directly the notations and conventions defined in [14, 16, 23] without further explanations. Then, the field equations are given in Appendix A.

¹ In particular, we have $\varphi = -\nu^{HMT}$, $K_{ij} = -K_{ij}^{HMT}$, $\mathcal{A} = a^{HMT}$, $\Lambda_g = \Omega^{HMT}$, $\mathcal{G}_{ij} = \Theta_{ij}^{HMT}$, where quantities with super indice ‘‘HMT’’ are the ones used in [13].

III. COSMOLOGICAL PERTURBATIONS

In this section, we first give a brief review of the FRW universe, and then argue that it must be flat in the framework of the HMT generalization. This is a very important implication. In fact, one of the main motivations of inflation was to solve the flatness problem [25]. In the second part of this section, we consider scalar perturbations without restricting ourselves to a particular gauge. We shall closely follow the presentation given in [16], which will be referred to as Paper I. To see the differences, we present our formulas closely parallel to those given in GR [24], and point out the similarities and differences whenever they raise.

A. Flatness of the FRW Universe

The homogeneous and isotropic universe is described by,

$$\bar{N} = 1, \quad \bar{N}_i = 0, \quad \bar{g}_{ij} = a^2(t) \gamma_{ij}, \quad (3.1)$$

where $\gamma_{ij} = \delta_{ij} (1 + \frac{1}{4} \kappa r^2)^{-2}$, with $r^2 \equiv x^2 + y^2 + z^2$, $\kappa = 0, \pm 1$. As in Paper I, we use symbols with bars to denote the quantities of the background in the (t, x, y, z) -coordinates. Using the $U(1)$ gauge freedom of Eq.(1.7), on the other hand, we can set

$$\bar{\varphi} = 0. \quad (3.2)$$

Then, we find

$$\begin{aligned} \bar{K}_{ij} &= -a^2 H \gamma_{ij}, \quad \bar{R}_{ij} = 2\kappa \gamma_{ij}, \\ \bar{F}_A^{ij} &= \frac{2\kappa \bar{A}}{a^4} \gamma^{ij}, \quad \bar{F}_\varphi^{ij} = 0, \quad \bar{F}_\varphi^i = 0, \\ \bar{F}^{ij} &= \frac{\gamma^{ij}}{a^2} \left(-\Lambda + \frac{\kappa}{a^2} + \frac{2\Delta_1 \kappa^2}{a^4} + \frac{12\Delta_2 \kappa^3}{a^6} \right), \end{aligned} \quad (3.3)$$

and

$$\begin{aligned} \bar{\mathcal{L}}_K &= 3(1 - 3\lambda) H^2, \quad \bar{\mathcal{L}}_\varphi = 0 = \bar{\mathcal{L}}_\lambda, \\ \bar{\mathcal{L}}_A &= 2\bar{A} \left(\Lambda_g - \frac{3\kappa}{a^2} \right), \\ \bar{\mathcal{L}}_V &= 2\Lambda - \frac{6\kappa}{a^2} + \frac{12\Delta_1 \kappa^2}{a^4} + \frac{24\Delta_2 \kappa^3}{a^6}, \end{aligned} \quad (3.4)$$

where $H = \dot{a}/a$ and

$$\Delta_1 \equiv \frac{3g_2 + g_3}{\zeta^2}, \quad \Delta_2 \equiv \frac{9g_4 + 3g_5 + g_6}{\zeta^4}. \quad (3.5)$$

The super-momentum constraint (A.2) is satisfied identically, provided that $\bar{J}^i = 0$, while the Hamiltonian con-

straint (A.1) yields ²,

$$\frac{1}{2}(3\lambda-1)H^2 + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\bar{\rho} + \frac{\Lambda}{3} + \frac{2\Delta_1\kappa^2}{a^4} + \frac{4\Delta_2\kappa^3}{a^6}, \quad (3.6)$$

where $\bar{J}^t \equiv -2\bar{\rho}$. On the other hand, Eqs.(A.4) and (A.5) yield, respectively,

$$H\left(\Lambda_g - \frac{\kappa}{a^2}\right) = -\frac{8\pi G}{3}\bar{J}_\varphi, \quad (3.7)$$

$$\frac{3\kappa}{a^2} - \Lambda_g = 4\pi G\bar{J}_A, \quad (3.8)$$

while the dynamical equation (A.7) reduces to

$$\begin{aligned} \frac{1}{2}(3\lambda-1)\frac{\ddot{a}}{a} = & -\frac{4\pi G}{3}(\bar{\rho} + 3\bar{p}) + \frac{1}{3}\Lambda - \frac{2\Delta_1\kappa^2}{a^4} \\ & - \frac{8\Delta_2\kappa^3}{a^6} + \frac{1}{2}\bar{A}\left(\frac{\kappa}{a^2} - \Lambda_g\right), \end{aligned} \quad (3.9)$$

where $\bar{\tau}_{ij} = \bar{p}\bar{g}_{ij}$.

The conservation law of the momentum (A.14) is satisfied identically, while the one of the energy (A.13) reduces to,

$$\dot{\bar{\rho}} + 3H(\bar{\rho} + \bar{p}) = \bar{A}\bar{J}_\varphi. \quad (3.10)$$

From Eqs.(3.7) and (3.8), one can see that when

$$\bar{J}_A = 0 = \bar{J}_\varphi, \quad (3.11)$$

the universe is necessarily flat, $k = 0 = \Lambda_g$. This is true for the case where the source is a scalar field [16], as can be seen from Eqs.(B.12) and (B.13) given in the next section, where both \bar{J}_A and \bar{J}_φ are proportional to the spatial gradients of the scalar field χ . This can be easily generalized to the case with multi-scalar fields.

In general, the coupling of the gauge field A and the Newtonian prepotential φ to a matter field ψ_n is given by [15],

$$\int dt d^3x \sqrt{g} Z(\psi_n, g_{ij}, \nabla_k)(A - \mathcal{A}), \quad (3.12)$$

where \mathcal{A} is defined as

$$\mathcal{A} \equiv -\dot{\varphi} + N^i \nabla_i \varphi + \frac{1}{2}N(\nabla\varphi)^2, \quad (3.13)$$

and Z is the most general scalar operator under the full symmetry of Eq.(1.5), with its dimension

$$[Z] = 2. \quad (3.14)$$

For a vector field (A_0, A_i) , we have $[A_0] = 2$, $[A_i] = 0$ [28]. Then, we find

$$Z(A_0, A_i, g_{ij}, \nabla_k) = \mathcal{K} B_i B^i, \quad (3.15)$$

where \mathcal{K} is an arbitrary function of $A^i A_i$, and

$$B_i = \frac{1}{2} \frac{\varepsilon_i^{jk}}{\sqrt{g}} \mathcal{F}_{jk}, \quad \nabla^i B_i = 0, \quad (3.16)$$

with $\mathcal{F}_{ij} \equiv \partial_j A_i - \partial_i A_j$. This can be easily generalized to several vector fields, $(A_0^{(n)}, A_i^{(n)})$, for which we have

$$Z(\vec{A}_0, \vec{A}_i, g_{ij}, \nabla_k) = \sum_{m,n} \mathcal{K}_{mn} B_i^{(m)} B^{(n)i}, \quad (3.17)$$

where $\mathcal{K}_{m,n}$ is an arbitrary function of $A^{(k)i} A_i^{(l)}$. Then, it is easy to show that in the FRW background, we have $\bar{J}_A = 0$, because $\bar{B}_i^{(m)} = 0$ [30], as can be seen from Eq.(3.16). With the gauge choice (3.2), one can also show that $\bar{J}_\varphi = 0$. Therefore, an early universe dominated by vector fields is also necessarily flat. This can be further generalized to the case of Yang-Mills fields [31]. For fermions, on the other hand, their dimensions are $[\psi_n] = 3/2$ [32]. Then, $Z(\psi_n, g_{ij}, \nabla_k)$ cannot be a functional of ψ_n . Therefore, in this case \bar{J}_A and \bar{J}_φ vanish identically.

Although we cannot exhaust all the matter fields, with the special form of the coupling given by Eq.(3.12), it is quite reasonable to argue that *the universe is necessarily flat for all cosmologically viable models in the HMT setup*. Therefore, in the rest of this paper, we shall consider only the flat FRW universe, i.e.,

$$\kappa = 0 = \Lambda_g, \quad (3.18)$$

for which Eq.(3.11) holds.

B. Linear Perturbations

As mentioned previously, to solve the strong coupling problem, one needs to impose the condition (1.8). Once it is satisfied, one can safely carry out the linear perturbations. With this in mind, as usual, we study these perturbations in terms of the conformal time η , where $\eta = \int dt/a(t)$. Under this coordinate transformation, the fields transform as,

$$\begin{aligned} N &= a\tilde{N}, \quad N^i = a\tilde{N}^i, \quad g_{ij} = \tilde{g}_{ij}, \\ A &= a\tilde{A}, \quad \varphi = \tilde{\varphi}, \end{aligned} \quad (3.19)$$

where the quantities with tildes are the ones defined in the coordinates (t, x^i) . With these in mind, we write the linear scalar perturbations of the metric in the form,

$$\begin{aligned} \delta N &= a\phi, \quad \delta N_i = a^2 B_{,i}, \\ \delta g_{ij} &= -2a^2(\psi\delta_{ij} - E_{,ij}), \\ A &= \hat{A} + \delta A, \quad \varphi = \hat{\varphi} + \delta\varphi, \end{aligned} \quad (3.20)$$

² Since now the Hamiltonian constraint is a global one, one can include a “dark matter component as an integration constant,” as first noted in [29]. For the sake of simplicity, in this paper we shall not consider this possibility, and it is not difficult to show that our mainly conclusions are equally applicable to this case.

where $\hat{A} = a\bar{A}$ and $\hat{\varphi} = \bar{\varphi}$. Quantities with hats denote the ones of the background in the coordinates (η, x^i) . Under the gauge transformations (1.2), they transform as

$$\begin{aligned}\tilde{\phi} &= \phi - \mathcal{H}\xi^0 - \xi^{0'}, & \tilde{\psi} &= \psi + \mathcal{H}\xi^0, \\ \tilde{B} &= B + \xi^0 - \xi', & \tilde{E} &= E - \xi, \\ \widetilde{\delta\varphi} &= \delta\varphi - \xi^0\hat{\varphi}', & \widetilde{\delta A} &= \delta A - \xi^0\hat{A}' - \xi^{0'}\hat{A},\end{aligned}\quad (3.21)$$

where $f = -\xi^0$, $\zeta^i = -\xi^{,i}$, $\mathcal{H} \equiv a'/a$, and a prime denotes the ordinary derivative with respect to η . Under the $U(1)$ gauge transformations, on the other hand, we find that

$$\begin{aligned}\tilde{\phi} &= \phi, & \tilde{E} &= E, & \tilde{\psi} &= \psi, & \tilde{B} &= B - \frac{\epsilon}{a}, \\ \widetilde{\delta\varphi} &= \delta\varphi + \epsilon, & \widetilde{\delta A} &= \delta A - \epsilon',\end{aligned}\quad (3.22)$$

where $\epsilon = -\alpha$. Then, the gauge transformations of the whole group $U(1) \ltimes \text{Diff}(M, \mathcal{F})$ will be the linear combination of the above two. Out of the six unknowns, one can construct three gauge-invariant quantities [16],

$$\begin{aligned}\Phi &= \phi - \frac{1}{a - \hat{\varphi}'}(a\sigma - \delta\varphi)' \\ &\quad - \frac{1}{(a - \hat{\varphi}')^2}(\hat{\varphi}'' - \mathcal{H}\hat{\varphi}') (a\sigma - \delta\varphi), \\ \Psi &= \psi + \frac{\mathcal{H}}{a - \hat{\varphi}'}(a\sigma - \delta\varphi), \\ \Gamma &= \delta A + \left[\frac{a(\delta\varphi - \hat{\varphi}'\sigma) - \hat{A}(a\sigma - \delta\varphi)}{a - \hat{\varphi}'} \right]',\end{aligned}\quad (3.23)$$

where $\sigma \equiv E' - B$. For the background, we have chosen the gauge (3.2), for which Eq.(3.23) reduces to

$$\begin{aligned}\Phi &= \phi - \frac{1}{a}(a\sigma - \delta\varphi)', \\ \Psi &= \psi - \frac{\mathcal{H}}{a}(\delta\varphi - a\sigma), \\ \Gamma &= \delta A + \left[\delta\varphi - \frac{\hat{A}}{a}(a\sigma - \delta\varphi) \right]', \quad (\bar{\varphi} = 0).\end{aligned}\quad (3.24)$$

Then, for the general perturbations (3.20), we have

$$\begin{aligned}\delta K_{ij} &= a \left\{ \psi' \delta_{ij} - \sigma_{,ij} \right. \\ &\quad \left. + \mathcal{H} \left[(2\psi + \phi) \delta_{ij} - 2E_{,ij} \right] \right\}, \\ \delta R_{ij} &= \psi_{,ij} + \partial^2 \psi \delta_{ij}.\end{aligned}\quad (3.25)$$

Thus, to first-order the Hamiltonian and momentum constraints become, respectively,

$$\begin{aligned}\int d^3x \left\{ \partial^2 \psi - \frac{1}{2}(3\lambda - 1)\mathcal{H} \left[3(\psi' + \mathcal{H}\phi) - \partial^2 \sigma \right] \right. \\ \left. - 4\pi G a^2 \delta\mu \right\} = 0,\end{aligned}\quad (3.26)$$

$$\begin{aligned}(3\lambda - 1)(\psi' + \mathcal{H}\phi) + (1 - \lambda)\partial^2 \left(\sigma - \frac{1}{a}\delta\varphi \right) \\ = 8\pi G a q + \Delta(\eta),\end{aligned}\quad (3.27)$$

where

$$\delta\mu \equiv -\frac{1}{2}\delta J^t, \quad \delta J^i \equiv \frac{1}{a^2}q^i, \quad (3.28)$$

$q^i = \delta^{ij}q_{,j}$, and $\Delta(\eta)$ is an integration function. In GR, it is usually set to zero [24]. However, in the present case, since $\phi = \phi(\eta)$, another interesting choice is $\Delta(\eta) = (3\lambda - 1)\mathcal{H}\phi$, which will cancel the second term in the left-hand side of Eq.(3.27).

On the other hand, the linearized equations (A.4) and (A.5) reduce, respectively, to

$$\begin{aligned}2\mathcal{H}\partial^2\psi + (1 - \lambda)\partial^2 \left[3(\psi' + \mathcal{H}\phi) - \frac{1}{a}\partial^2(a\sigma - \delta\varphi) \right] \\ = 8\pi G a^3 \delta J_\varphi,\end{aligned}\quad (3.29)$$

$$\partial^2\psi = 2\pi G a^2 \delta J_A, \quad (3.30)$$

while the linearly perturbed dynamical equations can be divided into the trace and traceless parts. The trace part reads,

$$\begin{aligned}\psi'' + 2\mathcal{H}\psi' + \mathcal{H}\phi' + (2\mathcal{H}' + \mathcal{H}^2)\phi \\ - \frac{1}{3}\partial^2(\sigma' + 2\mathcal{H}\sigma) \\ - \frac{2}{3(3\lambda - 1)} \left(1 + \frac{\alpha_1}{a^2}\partial^2 + \frac{\alpha_2}{a^4}\partial^4 \right) \partial^2\psi \\ - \frac{\Lambda_g a}{3(3\lambda - 1)} 2\hat{A}(\partial^2 E - 3\psi - 3\phi/2) \\ - \frac{\Lambda_g a}{3(3\lambda - 1)} \left[2\hat{A}(3\psi - \partial^2 E) + 3(\delta\varphi' + \delta A) \right] \\ + \frac{2}{3(3\lambda - 1)a} \partial^2(\hat{A}\psi - \delta A + \mathcal{H}\delta\varphi) \\ + \frac{\lambda - 1}{(3\lambda - 1)a} \partial^2(\delta\varphi' + \mathcal{H}\delta\varphi) = \frac{8\pi G a^2}{3\lambda - 1} \delta p,\end{aligned}\quad (3.31)$$

where

$$\begin{aligned}\alpha_1 &\equiv \frac{8g_2 + 3g_3}{\zeta^2}, \quad \alpha_2 \equiv \frac{8g_7 - 3g_8}{\zeta^4}, \\ \delta p &\equiv \delta\mathcal{P} + \frac{2}{3}\bar{p}\partial^2 E, \quad \Pi^{GR} \equiv \Pi + 2\bar{p}E, \\ \delta\tau^{ij} &= \frac{1}{a^2} \left[(\delta\mathcal{P} + 2\bar{p}\psi) \delta^{ij} + \Pi^{<ij>} \right], \\ \Pi^{<ij>} &= \Pi^{,ij} - \frac{1}{3}\delta^{ij}\partial^2 \Pi.\end{aligned}\quad (3.32)$$

The traceless part is given by

$$\begin{aligned}\psi - \phi + \sigma' + 2\mathcal{H}\sigma + \frac{1}{a^2} \left(\alpha_1 + \frac{\alpha_2}{a^2}\partial^2 \right) \partial^2\psi \\ - \frac{1}{a} \left[\hat{A}\psi - (\delta A - \mathcal{H}\delta\varphi) \right] = 8\pi G a^2 \Pi^{GR} + G(\eta),\end{aligned}\quad (3.33)$$

where $G(\eta)$ is another integration function. Again, in GR it is set to zero [24]. But, similar to the momentum constraint (3.27), one can also choose $G(\eta) = -\phi$ so that the second term in the left-hand side of the above equation is canceled.

The conservation laws (A.13) and (A.14) to first order are given, respectively, by,

$$\int d^3x \left\{ 2a \left[\delta\mu' + 3\mathcal{H}(\delta\mathcal{P} + \delta\mu) + 2\bar{p}\mathcal{H}\delta^2 E \right. \right. \\ \left. \left. + (\bar{p} + \bar{p}) (\partial^2 E - 3\psi)' - \bar{J}_\varphi \delta\varphi' \right] \right. \\ \left. - \hat{A}(\delta J_A' + 3\mathcal{H}\delta J_A) - 3\mathcal{H}\bar{J}_A(\delta A - \hat{A}\phi) \right. \\ \left. + \hat{A}\bar{J}_A(3\psi - \partial^2 E)' \right\} = 0, \quad (3.34)$$

$$q' + 3\mathcal{H}q - a\delta p - \frac{2a}{3}\partial^2 \Pi^{GR} = I(\eta), \quad (3.35)$$

where $I(\eta)$ is another integration function of η only. In GR, it is usually chosen to be zero [24].

This completes the general description of linear scalar perturbations in the flat FRW background in the framework of the HMT setup with any given λ [15], without choosing any specific gauge for the linear perturbations. However, before closing this section, let us consider some possible gauges.

C. Gauge Choices

To consider the gauge choices, we first note that

$$\xi = \xi(\eta, x), \quad \xi^0 = \xi^0(\eta).$$

Then, from Eqs.(3.21) and (3.22) one immediately finds that *the spatially flat gauge* $\tilde{\psi} = 0 = \tilde{E}$ [24] is impossible in the HMT setup. Since $\phi = \phi(\eta)$, a natural gauge for the time sector is

$$\tilde{\phi} = 0, \quad (3.36)$$

for which ξ^0 is uniquely fixed up to a constant C ,

$$\xi^0(\eta) = \frac{1}{a(\eta)} \int^\eta a(\eta') \phi(\eta') d\eta' + \frac{C}{a(\eta)}. \quad (3.37)$$

Then, depending on the choices of ξ and ϵ , we can have various different gauges.

1. Longitudinal Gauge

The longitudinal gauge in GR is defined as [24],

$$\tilde{E} = 0 = \tilde{B}, \quad (3.38)$$

which is impossible in the HL theory without the U(1) symmetry [23]. However, with the U(1) gauge freedom,

Eqs.(3.21) and (3.22) show that now this gauge becomes possible with the choice,

$$\xi = E, \quad \epsilon = a(B - E' + \xi^0), \quad (3.39)$$

where ξ^0 is given by Eq.(3.37). It should be noted that this gauge is fundamentally different from that given in GR [24], because now we also have $\tilde{\phi} = 0$.

2. Synchronous Gauge

In GR, the synchronous gauge is defined as [24],

$$\tilde{\phi} = 0 = \tilde{B}. \quad (3.40)$$

However, this is already implied in the above longitudinal gauge. With the extra U(1) gauge freedom ϵ , we can further require,

$$(i) \tilde{\delta\varphi} = 0, \quad \text{or} \quad (ii) \tilde{\delta A} = 0. \quad (3.41)$$

The former will be referred to as *the Newtonian synchronous gauge*, while the latter *the Maxwell synchronous gauge*. For the Newtonian synchronous gauge, ϵ and ξ are given by

$$\xi(\eta, x) = \int^\eta \left[B + \xi^0 + \frac{1}{a} (\delta\varphi - \xi^0 \dot{\varphi}') \right] d\eta' + D(x), \\ \epsilon(\eta, x) = \xi^0 \dot{\varphi}' - \delta\varphi, \quad (3.42)$$

where $D(x)$ is an arbitrary function of x^i only. For the Maxwell synchronous gauge, they are given by

$$\epsilon(\eta, x) = \int^\eta \left[\delta A - (\xi^0 \hat{A})' \right] d\eta' + D_1(x), \\ \xi(\eta, x) = \int^\eta \left(B + \xi^0 - \frac{\epsilon}{a} \right) d\eta' + D_2(x), \quad (3.43)$$

where $D_1(x)$ and $D_2(x)$ are other two arbitrary functions of x^i only. From the above one can see that none of them can fix the gauge uniquely.

3. Quasilongitudinal Gauge

In [14, 16], the gauge,

$$\tilde{\phi} = \tilde{E} = \tilde{\delta\varphi} = 0, \quad (3.44)$$

was used. With this gauge, we find that

$$\xi(\eta, x) = E(\eta, x), \quad \epsilon(\eta, x) = \xi^0 \dot{\varphi}' - \delta\varphi(\eta, x), \quad (3.45)$$

and ξ^0 is given by Eq.(3.37). Thus, in this case the gauge freedom of Eqs.(3.21) and (3.22) are also uniquely determined up to the constant C , similar to the longitudinal gauge (3.38).

Note that instead of choosing the above gauge, one can also choose

$$\tilde{\phi} = \tilde{E} = \tilde{\delta A} = 0, \quad (3.46)$$

for which we have

$$\begin{aligned} \xi(\eta, x) &= E(\eta, x), \\ \epsilon(\eta, x) &= \int^\eta \left[\delta A - \left(\xi^0 \hat{A} \right)' \right] d\eta' + D_3(x), \end{aligned} \quad (3.47)$$

where $D_3(x)$ is another integration function of x^i only. Thus, unlike the gauge (3.44), now the gauge is fixed only up to a constant C and an arbitrary function $D_3(x)$.

To be distinguish from the one defined in the case without the U(1) symmetry [23], we shall refer the gauge (3.44) to as *the Newtonian quasilongitudinal gauge*, and Eq.(3.46) *the Maxwell quasilongitudinal gauge*.

IV. INFLATION OF A SCALAR FIELD

In Appendix B, we construct the action for a single scalar field. In this section, we apply the perturbations developed in Sec. III to study inflationary models of such a scalar field. To this goal, let us first consider the slow-roll conditions.

A. Slow-Roll Inflation

For the flat FRW background, we find that

$$\begin{aligned} \bar{J}^t &= -2f \left(\frac{1}{2} \dot{\bar{\chi}}^2 + \tilde{V}(\bar{\chi}) \right) \equiv -2\bar{\rho}, \\ \bar{J}^i &= \bar{J}_\varphi = \bar{J}_A = 0, \\ \bar{\tau}_{ij} &= fa^2 \left(\frac{1}{2} \dot{\bar{\chi}}^2 - \tilde{V}(\bar{\chi}) \right) \delta_{ij} \equiv a^2 \bar{p} \delta_{ij}, \end{aligned} \quad (4.1)$$

where $\tilde{V}(\bar{\chi}) \equiv V(\bar{\chi})/f$. Then, Eqs.(3.6) - (3.8) and (3.10) yield $\Lambda_g = 0$ and

$$H^2 = \frac{8\pi\tilde{G}}{3} \left(\frac{1}{2} \dot{\bar{\chi}}^2 + \tilde{V}(\bar{\chi}) \right) + \frac{\tilde{\Lambda}}{3}, \quad (4.2)$$

where

$$\tilde{G} \equiv \frac{2fG}{3\lambda-1}, \quad \tilde{\Lambda} \equiv \frac{2\Lambda}{3\lambda-1}. \quad (4.3)$$

On the other hand, Eq.(B.17) reduces to,

$$\ddot{\bar{\chi}} + 3H\dot{\bar{\chi}} + \tilde{V}' = 0. \quad (4.4)$$

Eqs.(4.2) and (4.4) are identical to these given in GR [24], if one identifies \tilde{G} and $\tilde{\Lambda}$ to the Newtonian and cosmological constants, respectively. As a result, all the conditions

for inflationary models obtained in GR are equally applicable to the current case, as long as the background is concerned. In particular, the slow-roll conditions,

$$\tilde{\epsilon}_v, |\tilde{\eta}_v| \ll 1, \quad (4.5)$$

need to be imposed in order to get enough e-fold, where

$$\begin{aligned} \tilde{\epsilon}_v &\equiv \frac{\tilde{M}_{\text{pl}}^2}{2} \left(\frac{\tilde{V}'}{\tilde{V}} \right)^2 = \frac{3\lambda-1}{2f} \epsilon_v, \\ \tilde{\eta}_v &\equiv \tilde{M}_{\text{pl}}^2 \left(\frac{\tilde{V}''}{\tilde{V}} \right) = \frac{3\lambda-1}{2f} \eta_v, \end{aligned} \quad (4.6)$$

with $\tilde{M}_{\text{pl}}^2 \equiv 1/(8\pi\tilde{G})$, and ϵ_v and η_v are the ones defined in GR [25].

However, due to the presence of high-order spatial derivatives, the perturbations will be dramatically different, as to be shown below.

B. Linear Perturbations

In this section, in order for the formulas developed below to be applicable to as many cases as possible, we shall not restrict ourselves to any specific gauge. Then, to first-order we find that

$$\begin{aligned} \delta\rho &\equiv \delta\mu = \frac{f\bar{\chi}'}{a^2} (\delta\chi' - \bar{\chi}'\phi) + \frac{V_4}{a^4} \partial^4 \delta\chi + V' \delta\chi, \\ q &= \frac{f\bar{\chi}'}{a} \delta\chi, \quad \delta J_A = \frac{2c_1}{a^2} \partial^2 \delta\chi, \\ \delta J_\varphi &= \frac{1}{a^3} \left[(c_1' \bar{\chi}' + c_1 \mathcal{H} - f\bar{\chi}') \partial^2 \delta\chi + c_1 \partial^2 \delta\chi' \right], \\ \delta p &= \frac{f\bar{\chi}'}{a^2} (\delta\chi' - \bar{\chi}'\phi) - V' \delta\chi, \\ \Pi &= -2\bar{p}E, \quad \Pi^{GR} = 0. \end{aligned} \quad (4.7)$$

Hence, Eqs. (3.26) - (3.30) read, respectively,

$$\begin{aligned} &\int d^3x \left\{ \partial^2 \psi - \frac{1}{2} (3\lambda-1) \mathcal{H} \left[3(\psi' + \mathcal{H}\phi) - \partial^2 \sigma \right] \right\} \\ &= \int d^3x 4\pi G \left\{ f\bar{\chi}' (\delta\chi' - \bar{\chi}'\phi) + \frac{V_4}{a^2} \partial^4 \delta\chi \right. \\ &\quad \left. + a^2 V' \delta\chi \right\}, \end{aligned} \quad (4.8)$$

$$\begin{aligned} &(3\lambda-1)\psi' + (1-\lambda)\partial^2 \left(\sigma - \frac{1}{a} \delta\varphi \right) \\ &= 8\pi G f \bar{\chi}' \delta\chi, \end{aligned} \quad (4.9)$$

$$\begin{aligned} &2\mathcal{H}\psi + (1-\lambda) \left[3\psi' - \frac{1}{a} \partial^2 (a\sigma - \delta\varphi) \right] \\ &= 8\pi G \left[(c_1' \bar{\chi}' + c_1 \mathcal{H} - f\bar{\chi}') \delta\chi + c_1 \delta\chi' \right], \end{aligned} \quad (4.10)$$

$$\psi = 4\pi G c_1 \delta\chi. \quad (4.11)$$

Note that in writing Eq.(4.9), we had chosen $\Delta(\eta) = (3\lambda - 1)\mathcal{H}\phi$. It is also interesting to note that, unlike the case without the U(1) symmetry [34], now the metric perturbation ψ is proportional to $\delta\chi$. It is this difference that leads to a master equation, as to be shown below. Without the U(1) symmetry, this is in general impossible [34].

The trace and traceless parts of dynamical equation read, respectively,

$$\begin{aligned} & \psi'' + 2\mathcal{H}\psi' + \mathcal{H}\phi' + (2\mathcal{H}' + \mathcal{H}^2)\phi - \frac{1}{3}\partial^2(\sigma' + 2\mathcal{H}\sigma) \\ & - \frac{2}{3(3\lambda - 1)}\left(1 + \frac{\alpha_1}{a^2}\partial^2 + \frac{\alpha_2}{a^4}\partial^4\right)\partial^2\psi \\ & + \frac{2}{3(3\lambda - 1)a}\partial^2(\hat{A}\psi - \delta A + \mathcal{H}\delta\varphi) \\ & + \frac{\lambda - 1}{(3\lambda - 1)a}\partial^2(\delta\varphi' + \mathcal{H}\delta\varphi) \\ & = \frac{8\pi G}{3\lambda - 1}\left[f\bar{\chi}'(\delta\chi' - \bar{\chi}'\phi) - a^2V'\delta\chi\right], \end{aligned} \quad (4.12)$$

$$\begin{aligned} & \psi + \sigma' + 2\mathcal{H}\sigma + \frac{1}{a^2}\left(\alpha_1 + \frac{\alpha_2}{a^2}\partial^2\right)\partial^2\psi \\ & - \frac{1}{a}\left[\hat{A}\psi - (\delta A - \mathcal{H}\delta\varphi)\right] = 0, \end{aligned} \quad (4.13)$$

where in writing Eq.(4.13) we had set $G(\eta) = -\phi$. The energy conservation law now takes the form,

$$\begin{aligned} & \int d^3x a^2 \bar{\chi}' \left\{ f\delta\chi'' + 2\mathcal{H}f\delta\chi' + a^2V''\delta\chi + 2a^2\bar{V}'\phi \right. \\ & \left. - f\bar{\chi}'\left[\phi - (\partial^2 E - 3\psi)\right]' - \frac{\bar{A}(ac_1\delta\chi)'}{a} \right\} \\ & = - \int d^3x \partial^4 \left\{ V_4\delta\chi' + (V_4'\bar{\chi}' - V_4\mathcal{H})\delta\chi \right\}, \end{aligned} \quad (4.14)$$

The momentum conservation is identically satisfied, while the Klein-Gordon equation becomes

$$\begin{aligned} & f\left\{ \delta\chi'' + 2\mathcal{H}\delta\chi' - \bar{\chi}'[3\psi' + \phi' - \partial^2\sigma] \right\} \\ & + 2a^2V'\phi + (a^2V'' - \partial^2)\delta\chi \\ & = \frac{\partial^2}{a}\left[2\hat{A}(c_1' - c_2)\delta\chi - c_1\delta\varphi' + f\bar{\chi}'\delta\varphi + c_1\delta A\right] \\ & + 2\left(V_1 - \frac{V_2 + V_4'}{a^2}\partial^2 - \frac{V_6}{a^4}\partial^4\right)\partial^2\delta\chi, \end{aligned} \quad (4.15)$$

which can be rewritten as a perturbed energy balance equation,

$$\begin{aligned} & \delta\rho' + 3\mathcal{H}(\delta\rho + \delta p) \\ & - (\bar{\rho} + \bar{p})\left(3\psi' - \partial^2\sigma - \frac{1}{f}\partial^2(v + B)\right) \\ & = \frac{1}{f}(\bar{\rho} + \bar{p})\delta Q^{\text{HMT}}, \end{aligned} \quad (4.16)$$

where

$$\begin{aligned} \delta Q^{\text{HMT}} &= \frac{V_4}{a^2\bar{\chi}'^2}\partial^4\delta\chi' + \frac{1}{\bar{\chi}'}\left[2V_1 - 2\frac{V_6}{a^4}\partial^4\right. \\ & \left. - \frac{1}{a^2}\left(2V_2 + V_4' + \frac{V_4\mathcal{H}}{\bar{\chi}'}\right)\partial^2\right]\partial^2\delta\chi \\ & + \frac{1}{a\bar{\chi}'}\partial^2\left[2\hat{A}(c_1' - c_2)\delta\chi - c_1\delta\varphi' \right. \\ & \left. + f\bar{\chi}'\delta\varphi + c_1\delta A\right], \\ q &\equiv -a(\bar{\rho} + \bar{p})(v + B). \end{aligned} \quad (4.17)$$

C. Uniform Density Perturbation

Under the gauge transformations (3.21) and (3.22), $\delta\chi$ and $\delta\rho$ transform, respectively, as

$$\widetilde{\delta\chi} = \delta\chi - \xi^0\bar{\chi}', \quad \widetilde{\delta\rho} = \delta\rho - \xi^0\bar{\rho}'. \quad (4.18)$$

Therefore, the quantity ζ defined by

$$-\zeta \equiv \psi + \frac{\mathcal{H}}{\bar{\rho}'}\delta\rho, \quad (4.19)$$

is gauge-invariant. In GR it is often referred to as the gauge-invariant perturbation on uniform-density hypersurfaces. It can be shown that it obeys the evolution equation,

$$\zeta' = -\frac{\mathcal{H}\delta p_{\text{nad}}}{\bar{\rho} + \bar{p}} + \frac{1}{3}\left[\delta Q^{\text{HMT}} - \partial^2\sigma - \frac{\partial^2}{f}(v + B)\right], \quad (4.20)$$

where the non-adiabatic pressure perturbation is defined as

$$\delta p_{\text{nad}} \equiv \delta p - \frac{\bar{p}'}{\bar{\rho}'}\delta\rho = \delta p_{\text{nad}}^{\text{GR}} + \delta p_{\text{nad}}^{\text{HMT}}, \quad (4.21)$$

with,

$$\begin{aligned} \delta p_{\text{nad}}^{\text{GR}} &\equiv \frac{2}{3a^2}\left(2 + \frac{\bar{\chi}''}{\mathcal{H}\bar{\chi}'}\right)\left[\bar{\chi}'(\delta\chi' - \bar{\chi}'\phi) \right. \\ & \left. - (\bar{\chi}'' - \mathcal{H}\bar{\chi}')\delta\chi\right] \\ &= -\frac{2a\bar{V}'}{3\mathcal{H}\bar{\chi}'}\left[\left(\frac{\bar{\chi}'}{a}\right)(\delta\chi' - \bar{\chi}'\phi) - \left(\frac{\bar{\chi}'}{a}\right)'\delta\chi\right], \\ \delta p_{\text{nad}}^{\text{HMT}} &\equiv \left(1 + \frac{2\bar{\chi}''}{\mathcal{H}\bar{\chi}'}\right)\frac{V_4}{3a^4}\partial^4\delta\chi. \end{aligned} \quad (4.22)$$

Note that Eq.(4.20) is quite similar to that of the case without the U(1) symmetry [34], and the only difference is the inclusion of the U(1) gauge field A and Newtonian prepotential φ in δQ^{HMT} , as one can see from Eq.(4.17) given above and Eq.(4.3) given in [34]. But, these terms vanish in the super-horizon region. As a result, all the conclusions obtained in [34] in this region are equally applicable to the present case. In particular, the perturbations in this region are adiabatic during the slow-roll inflation, as in GR.

D. Comoving Curvature Perturbation

On the other hand, the comoving curvature perturbation, defined by

$$\mathcal{R} = \psi + \frac{\mathcal{H}}{\bar{\chi}'} \delta\chi, \quad (4.23)$$

is gauge-invariant even in the HL theory. From its definition, it can be shown that \mathcal{R} satisfies the equation,

$$\mathcal{R}' = \mathcal{H}\mathcal{S} + \frac{\mathcal{H}' - \mathcal{H}^2}{\bar{\chi}'} \delta\chi + \psi' + \mathcal{H}\phi, \quad (4.24)$$

where the dimensionless intrinsic entropy perturbation \mathcal{S} is defined as

$$\mathcal{S} \equiv \frac{\delta\chi' - \bar{\chi}'\phi}{\bar{\chi}'} - \frac{\bar{\chi}'' - \mathcal{H}\bar{\chi}'}{\bar{\chi}'^2} \delta\chi = -\frac{3\mathcal{H}}{2\bar{V}'\bar{\chi}'} \delta p_{nad}^{\text{GR}}, \quad (4.25)$$

where to get the last step Eq.(4.22) was used. In terms of \mathcal{R} the super-momentum constraint (4.9) can be written in the form,

$$\mathcal{R}' = \mathcal{H}\mathcal{S} + \frac{\lambda - 1}{3\lambda - 1} \partial^2 \left(\sigma - \frac{\delta\varphi}{a} \right), \quad (4.26)$$

which reduces to $\mathcal{R}' = \mathcal{H}\mathcal{S}$ on all scales in the relativistic limit $\lambda \rightarrow 1$. Thus, in the slow-roll approximations and neglecting the spatial gradients on large scales, we obtain the same conclusion as that given in [34], namely, the comoving curvature perturbation has two modes on large scales, a constant mode and a rapidly decaying mode, given by

$$\mathcal{R} \simeq C_1 + C_2 \int \frac{d\eta}{a^2}. \quad (4.27)$$

In addition, unlike that in GR where the local Hamiltonian constraint enforces adiabaticity on large scales, in HMT setup it is the slow-roll evolution ($\ddot{\bar{\chi}} = 0$, or, $\bar{\chi}'' = \mathcal{H}\bar{\chi}'$) that leads to rapidly decaying entropy perturbations at late times.

Note that we could also find the first-order equation for \mathcal{S} by using the Klein-Gordon equation, which can be written in the form,

$$\begin{aligned} \mathcal{S}' + \left(2\frac{\bar{\chi}''}{\bar{\chi}'} + \mathcal{H} \right) \mathcal{S} &= \frac{1}{f\bar{\chi}'} \left\{ [f\bar{\chi}'3\mathcal{H}\phi] \right. \\ &+ \partial^2 \left[(1 + 2V_1) \delta\chi - \frac{2f\bar{\chi}'}{3\lambda - 1} \left(\sigma - \frac{\delta\varphi}{a} \right) \right] \\ &+ \frac{\partial^2}{a} \left[2\hat{A}(c_1' - c_2) \delta\chi + c_1 \delta A - c_1 \delta\varphi' \right] \\ &\left. - 2\frac{\partial^4}{a^2} \left[(V_2 + V_4') + \frac{V_6}{a^2} \partial^2 \right] \delta\chi \right\}. \end{aligned} \quad (4.28)$$

Thus, in the large scales (neglecting all the spatial gradient terms), the first term in the right-hand side is function of η only (Recall that $\phi = \phi(\eta)$). Then, the corresponding entropy equation depends only on time on these large-scales.

V. SCALAR PERTURBATIONS IN SUB- AND SUPER-HORIZON SCALES

So far, we have not chosen any gauge. In this section, we shall restrict ourselves to the Newtonian quasilonitudinal gauge defined by Eq.(3.44) in Sec. III.C, i.e.,

$$\phi = E = \delta\varphi = 0. \quad (5.1)$$

Then, Eqs.(4.8) - (4.15) can be cast in the forms of Eqs.(B.19) - (B.26). From Eqs.(B.21), (B.23), (B.25), we can express ψ , B and δA in terms of $\delta\chi$, and then submit them into Eq.(B.26), we obtain a master equation for $\delta\chi$, which can be written as

$$\delta\chi'' + \mathcal{P}\delta\chi' + \mathcal{Q}\delta\chi = \mathcal{F}\partial^2\delta\chi, \quad (5.2)$$

where

$$\begin{aligned} \beta_0 &\equiv f + \frac{4\pi G c_1^2}{|c_\psi^2|}, \\ \mathcal{P} &\equiv \frac{1}{\beta_0} (\beta_0' + 2\mathcal{H}\beta_0), \\ \mathcal{Q} &\equiv \frac{1}{\beta_0} \left\{ a^2 V'' + \frac{4\pi G c_1 c_1'' \bar{\chi}'^2}{|c_\psi^2|} - \frac{8\pi G}{\lambda - 1} f \bar{\chi}'^2 (f - c_1') \right. \\ &\quad \left. - 4\pi G c_1 a^2 V' \left(3 + \frac{c_1'}{f|c_\psi^2|} - \frac{1}{|c_\psi^2|} \right) \right\}, \\ \mathcal{F} &\equiv \frac{1}{\beta_0} \left\{ 1 + 2V_1 + 2\bar{A}(c_1' - c_2) - 4\pi G c_1^2 (1 - \bar{A}) \right. \\ &\quad \left. - \frac{2}{a^2} (V_2 + V_4' + 2\pi G \alpha_1 c_1^2) \partial^2 \right. \\ &\quad \left. - \frac{2}{a^4} (V_6 + 2\pi G \alpha_2 c_1^2) \partial^4 \right\}, \end{aligned} \quad (5.3)$$

with

$$c_\psi^2 \equiv \frac{\lambda - 1}{1 - 3\lambda}. \quad (5.4)$$

Setting

$$\delta\chi = \exp \left(-\frac{1}{2} \int \mathcal{P} d\eta \right) u, \quad (5.5)$$

one can write Eq.(5.2) in the momentum space in the form,

$$u_k'' + \omega_k^2 u_k = 0, \quad (5.6)$$

where

$$\begin{aligned} \omega_k^2 &= k^2 \mathcal{F}_k - \frac{1}{4} (2\mathcal{P}' + \mathcal{P}^2 - 4\mathcal{Q}), \\ \mathcal{F}_k &\equiv \frac{1}{\beta_0} \left\{ 1 + 2V_1 + 2\bar{A}(c_1' - c_2) - 4\pi G c_1^2 (1 - \bar{A}) \right. \\ &\quad \left. + \frac{2k^2}{a^2} (V_2 + V_4' + 2\pi G \alpha_1 c_1^2) \right. \\ &\quad \left. - \frac{2k^4}{a^4} (V_6 + 2\pi G \alpha_2 c_1^2) \right\}. \end{aligned} \quad (5.7)$$

Note that the above hold only for $\lambda \neq 1$. When $\lambda = 1$, we have a first-order equation for $\delta\chi$

$$\delta\chi' + \frac{\bar{\chi}'}{c_1} (c_1' - f) \delta\chi = 0, \quad (5.8)$$

which has the general solution,

$$\delta\chi = \exp \left\{ \int \frac{\bar{\chi}'}{c_1} (f - c_1') d\eta \right\} \delta\chi_1(x), \quad (\lambda = 1), \quad (5.9)$$

where $\delta\chi_1(x)$ is an arbitrary function of x only. Since in this paper we are mainly interested in the case $\lambda \neq 1$, in the following we shall not consider this case further.

Also, for the field to be stable in the UV regime, the condition

$$V_6 + 2\pi G \alpha_2 c_1^2 < 0, \quad (5.10)$$

has to be satisfied. To study Eq.(5.6) further, we consider the sub- and super-horizon scales, separately.

A. Sub-Horizon Scales

In this region, we have $k \gg \mathcal{H}$, and the dispersion relation reduces to,

$$\omega_k^2 \simeq -\frac{2k^6}{\beta_0 a^4} (V_6 + 2\pi G \alpha_2 c_1^2). \quad (5.11)$$

With the extreme slow-roll condition, we have $a \simeq -\frac{1}{H\eta}$ and $H, V_6, c_1 \simeq \text{Constants}$. Then, from Eq. (5.6) we find that

$$u_k \propto e^{i\omega_k \eta}. \quad (5.12)$$

Unlike the case without the U(1) symmetry [34], the metric perturbations ψ and B now oscillate with the same frequency as $\delta\chi$, as one can see from Eqs.(B.21) and (B.23). Therefore, they are always coupled to the scalar field modes.

B. Super-Horizon Scales

In this region, we have $k \ll \mathcal{H}$, and to the order of k^2 , we find that

$$\omega_k^2 \simeq \frac{k^2}{\beta_0} \left[1 + 2V_1 + 2\bar{A}(c_1' - c_2) - 4\pi G c_1^2 (1 - \bar{A}) \right] + \mathcal{Q} - \frac{2\mathcal{P}' + \mathcal{P}^2}{4}. \quad (5.13)$$

In the extreme slow-roll and massless limit ($\bar{\chi}' \simeq 0 \simeq V', V'' \simeq 0$), we obtain the following solution

$$u_k = -\frac{D_1}{H\eta} \left\{ 1 + \frac{k^2 \eta^2}{2\beta_0} \left[1 + 2V_1 \right. \right.$$

$$\left. \left. + 2\bar{A}(c_1' - c_2) - 4\pi G c_1^2 (1 - \bar{A}) \right] \right\} + D_2 \eta^2 \left\{ 1 - \frac{k^2 \eta^2}{10\beta_0} \left[1 + 2V_1 + 2\bar{A}(c_1' - c_2) - 4\pi G c_1^2 (1 - \bar{A}) \right] \right\} \sim D_1 a + D_2 \eta^2, \quad (5.14)$$

where the first term represents a constant perturbation, while the second term represents a decaying mode. Then, we find that

$$\begin{aligned} \delta\chi &\simeq D_1 - D_2 H \eta^3, \\ \psi &\simeq 4\pi G c_1 \delta\chi, \\ k^2 B &\simeq -\frac{12\pi G c_1}{|c_\psi|^2} D_2 \eta^2. \end{aligned} \quad (5.15)$$

In terms of the gauge-invariant quantities (3.24), we obtain

$$\begin{aligned} \Psi_k &= \psi_k - \mathcal{H} B_k, \\ \Phi_k &= \mathcal{H} B_k + B_k', \\ \Phi_k - \Psi_k &= H^2 k^2 \eta^2 (H^2 k^2 \eta^2 \alpha_2 - \alpha_1) \psi_k \\ &\quad + H \eta (\hat{A} \psi_k - \delta A_k). \end{aligned} \quad (5.16)$$

Thus, like in the case without the U(1) symmetry [34], the dynamical evolution now leads to $\Phi = \Psi \rightarrow 0$ at late times ($\eta \rightarrow 0$).

VI. POWER SPECTRA AND INDICES OF SCALAR AND TENSOR PERTURBATIONS

To calculate the spectra and indices of scalar and tensor perturbations with the slow-roll approximations, we shall use the uniform approximation, proposed recently in [26], and applied to the studies of tensor perturbations in the HL theory without the U(1) symmetry in [35, 36]. We shall closely follow the treatment presented in [36]. In particular, for perturbations given by,

$$v_k'' = [g(k, \eta) + q(\eta)] v_k, \quad (6.1)$$

where $q(\eta) = -1/4\eta^2$, and v_k is the canonically normalized field, the corresponding power spectrum and index at leading order of the uniform approximation are given as [36],

$$\begin{aligned} P_v(k)|_{k\eta \rightarrow 0^-} &\equiv \frac{k^3}{2\pi^2} |v_k|^2 \Big|_{k\eta \rightarrow 0^-} \\ &= \lim_{k\eta \rightarrow 0^-} \frac{k^3 \exp \left\{ 2\mathcal{D}(k, \eta) \right\}}{4\pi^2 a^2 \sqrt{g(k, \eta)}}, \end{aligned} \quad (6.2)$$

$$n_v - 1 \equiv \frac{d \ln P_v}{d \ln k} \Big|_{k\eta \rightarrow 0^-}, \quad (6.3)$$

where

$$\mathcal{D}(k, \eta) \equiv \int_{\bar{\eta}(k)}^{\eta} \sqrt{g(k, \eta')} d\eta', \quad (6.4)$$

and $\bar{\eta}(k)$ denotes the turning point $g(k, \bar{\eta}) = 0$. Note that in writing the above expressions, we assumed that there is only one turning point, that is, we consider only the case where $g(k, \eta) = 0$ has only one real root. For detail, see [36]. In the following, we shall apply the above to the cases of scalar and tensor perturbations.

A. Power Spectrum and Index of Scalar Perturbations

With the help of the master equation (5.2) and the definition of gauge-invariant \mathcal{R} in (4.23), the second order action reads,

$$S^{(2)} = \frac{1}{2} \int d\eta d^3x a^2 h^2 \left[\beta_0 \mathcal{R}'^2 - \beta_4 \mathcal{R}^2 - \beta_1 (\partial_i \mathcal{R})^2 - \beta_2 (\partial^2 \mathcal{R})^2 - \beta_3 (\partial_i \partial^2 \mathcal{R})^2 \right], \quad (6.5)$$

where

$$\begin{aligned} \beta_0 &= f + 4\pi G c_1^2 / |c_\psi^2|, \\ \beta_1 &\equiv [1 + 2V_1 + 2\bar{A}(c'_1 - c_2) - 4\pi G c_1^2 (1 - \bar{A})], \\ \beta_2 &\equiv \frac{2}{a^2} (V_2 + V_4' + 2\pi G c_1^2 \alpha_1), \\ \beta_3 &\equiv -\frac{2}{a^4} (V_6 + 2\pi G c_1^2 \alpha_2), \\ \beta_4 &\equiv \beta_0 \mathcal{Q} - \beta_0 \frac{h'^2}{h^2} + \frac{(a^2 \beta_0 h h')'}{a^2 h^2} \\ h &\equiv \left(4\pi G c_1 + \frac{H}{\dot{\chi}} \right)^{-1} = \frac{\delta\chi}{\mathcal{R}}. \end{aligned} \quad (6.6)$$

After introducing the variable

$$v \equiv z\mathcal{R}, \quad z^2 \equiv a^2 h^2 \beta_0, \quad (6.7)$$

the action is normalized to

$$S^{(2)} = \frac{1}{2} \int d\eta d^3x \left[(v')^2 - \frac{\beta_1}{\beta_0} (\partial_i v)^2 - m_{\text{eff}}^2 v^2 \right] - \frac{1}{2} \int d\eta d^3x \left[\frac{\beta_2}{\beta_0} (\partial^2 v)^2 + \frac{\beta_3}{\beta_0} (\partial_i \partial^2 v)^2 \right]. \quad (6.8)$$

Here m_{eff}^2 is defined to be

$$-m_{\text{eff}}^2 \equiv \frac{z''}{z} - \frac{\beta_4}{\beta_0}. \quad (6.9)$$

Going through the quantization procedure as described in Appendix C, the classical equation of motion for mode functions v_k are

$$v_k'' + (\omega_k^2 + m_{\text{eff}}^2) v_k = 0, \quad (6.10)$$

where

$$\omega_k^2 = \frac{k^2}{\beta_0} (\beta_1 + \beta_2 k^2 + \beta_3 k^4). \quad (6.11)$$

Looking at the expressions of the β coefficients, we see that they contain terms of $c_1, c_2, V_1, V_2, V_4, V_6$ and \bar{A} . Now go back to the Lagrangian describing the inflaton χ , Eqs.(B.2) and (B.9), one can see that V_1, V_2, V_4, V_6 all stem from the potential term \mathcal{V} , while c_1 and c_2 appear through the first line of (B.9), which can also be taken as a “potential term”. (Note that the second and the third line of (B.9) correspond to modifications of dynamical coupling terms due to the presence of φ .) Therefore, we could assign their respective “slow-roll” parameters describing their time evolution during inflation in a manner similar to V . However, unlike V , which appears in the background equation (4.2), these “potential terms” are not constrained by the background equations. As an approximation, we assume here that the time dependence of $c_1, c_2, V_1, V_2, V_4, V_6$ and \bar{A} are at least second order in terms of the slow roll parameters. Since we only consider the first order approximations in this paper, they can be taken as constant throughout inflation. This also leaves $d\beta_0/d\eta \propto dc_1/d\eta = 0$.

With the above assumptions, it can be shown that h^2 relating $\delta\chi$ and \mathcal{R} is of order $\mathcal{O}(\epsilon_v)$. In fact, from its definition,

$$\begin{aligned} h^2 &= \left(4\pi G c_1 + \frac{H}{\dot{\chi}} \right)^{-2} = \left(\frac{\dot{\chi}}{H} \right)^2 \left[1 + \frac{c_1 \dot{\chi}}{2M_{\text{pl}}^2 H} \right]^{-2} \\ &= 2\tilde{M}_{\text{pl}}^2 \epsilon_v \left[1 + \frac{c_1 \dot{\chi}}{2M_{\text{pl}}^2 H} \right]^{-2}. \end{aligned} \quad (6.12)$$

Since

$$\frac{c_1 \dot{\chi}}{2M_{\text{pl}}^2 H} = \frac{c_1}{\sqrt{2}M_{\text{pl}}} \times \frac{1}{\sqrt{2}M_{\text{pl}}} \frac{\dot{\chi}}{H} = \frac{c_1}{\sqrt{2}M_{\text{pl}}} \sqrt{\epsilon_v} \ll 1, \quad (6.13)$$

where $|c_1| \simeq M_* \ll M_{\text{pl}}$ [17], we have, to first order of the slow-roll parameters,

$$h^2 \simeq 2\tilde{M}_{\text{pl}}^2 \epsilon_v. \quad (6.14)$$

On the other hand, $a(\eta) \simeq -(1 + \epsilon_v)/(H\eta)$, which leads to

$$-m_{\text{eff}}^2 \simeq \frac{2 - 3\eta_v + 9\epsilon_v}{\eta^2} + \frac{\Delta m^2}{\eta^2}. \quad (6.15)$$

Here the first term comes from z''/z and is the same as that from GR under the above assumptions, whereas the second term introduces new effects,

$$\begin{aligned} \Delta m^2 &\simeq \frac{1}{\beta_0} \left[3(\beta_0 - 1)\eta_v + \left(\frac{2f^2}{\lambda - 1} - 6\beta_0 \right) \epsilon_v \right. \\ &\quad \left. - \frac{2}{\lambda - 1} \left(\frac{c_1}{\sqrt{2}M_{\text{pl}}} \sqrt{\epsilon_v} \right) \right]. \end{aligned} \quad (6.16)$$

We see that these are in general functions of λ . Since it's the time-dependence of m_{eff} which breaks the exact scale-invariance, we would expect that observations on the power index, which will be derived below, place constraints on the value of λ .

The function $g(k, \eta)$ defined through Eqs.(6.1) and (6.10) is now given by

$$g(k, \eta) = \frac{k^2}{y^2} [a_0^2 - (a_2 y^2 + a_4 y^4 + a_6 y^6)], \quad (6.17)$$

where

$$\begin{aligned} y &\equiv k\eta, \\ a_0^2 &\equiv \frac{1}{4} - m_{\text{eff}}^2 \eta^2 = \frac{9}{4} - 3\eta_V + 9\epsilon_V + \Delta m^2, \\ a_2 &\equiv \frac{1}{\beta_0} [1 + 2V_1 + 2\bar{A}(c_1' + c_2) - 4\pi G c_1^2 (1 - \bar{A})], \\ a_4 &\equiv \frac{1}{\beta_0} [2H^2(1 - 2\epsilon_V)(V_2 + V_4' + 2\pi G c_1^2 \alpha_1)], \\ a_6 &\equiv -\frac{1}{\beta_0} [2H^4(1 - 4\epsilon_V)(V_6 + 2\pi G c_1^2 \alpha_2)]. \end{aligned} \quad (6.18)$$

Thus the power spectrum of \mathcal{R} is given by,

$$\begin{aligned} P_{\mathcal{R}}(k) &= \frac{H^2(1 - 2\epsilon_V)|y_0|^3}{4\pi^2 \beta_0 a_0 h^2} \left(\frac{2}{e}\right)^{2a_0} \exp\left[\frac{a_4 y_0^4 + a_6 y_0^6}{3a_0}\right] \\ &\times \lim_{y \rightarrow 0} \left(\frac{y}{y_0}\right)^{3-2a_0}, \end{aligned} \quad (6.19)$$

where y_0 is defined to be the turning point of $g(k, \eta)$, namely,

$$a_6 y_0^6 + a_4 y_0^4 + a_2 y_0^2 - a_0^2 = 0. \quad (6.20)$$

Clearly, y_0 is independent of k , as a_n 's are. Then, we find

$$n_{\mathcal{R}} - 1 = 2\eta_V - 6\epsilon_V - \frac{2}{3}\Delta m^2. \quad (6.21)$$

Due to correction term Δm^2 , the spectrum index of the scalar perturbation does not reproduce the GR value in general. In particular, we see that it depends on the value $1/(\lambda - 1)$ (Note that β_0 is also a function of $\lambda - 1$). One may worry that in the relativistic limit at low energy $\lambda \rightarrow 1$ these terms will diverge, hence breaking the near-scale-invariance of the spectrum. However, during inflation, we are in a region where UV physics dominates, thus the value of λ is expected far away from its relativistic fixed point at that time.

We note that even when the index can restore the standard value in GR,³ it is a consequence of our assumption that all the potential terms c_1, c_2, V_1, V_2, V_4 and

V_6 are time-independent. Comparing the definition of z given here in Eq.(6.7) with the one given in GR, we can see some extra terms of c_1 appear. By our assumption, $dc_1/d\eta = 0$, this makes $d\beta_0/d\eta = 0$ and $dh^2/d\eta \propto d(\ddot{\chi}/H)/d\eta$, which leaves the term z''/z exactly the same as that of a single field in GR. Further more, in the modified dispersion relation (6.11), the term β_1/β_0 corresponds to the relativistic case, and the ones β_2/β_0 and β_3/β_0 are induced by Lorentz-symmetry-breaking effects, which are assumed to be time-independent. If, however, these potential terms are evolving with time during inflation, one needs to take into account of the time-dependence of the dispersion relation and of the varying effective mass [37]. The same arguments also apply to the studies of tensor spectrum and index.

We would also like to note that, the exact form of scale-dependence of the scalar spectrum depends on the instant when it's evaluated [38], and can receive further corrections when we incorporate a second order uniform approximation [26]. What's more, from an observational point of view, as long as the scale-dependence is not broken severely, the connection between tensor-scalar-ratio and slow-roll parameters is more important than the tilt itself.

Setting the slow roll parameters to zero exactly, the power spectrum given above can be put in the simple form,

$$\begin{aligned} P_{\mathcal{R}}(k) &= \frac{4H^4|y_0|^3}{3\pi^2 e^3 \beta_0 \dot{\chi}^2} \\ &\times \exp\left[\frac{2}{9}(a_4 y_0^4 + a_6 y_0^6)\right]. \end{aligned} \quad (6.22)$$

In the relativistic limit ($a_2 = \beta_0 = 1, a_4 = a_6 = c_1 = 0$, and $g_s = 0, (s = 2, \dots, 8)$), this yields the well-known result obtained in GR [25],

$$P_{\mathcal{R}}^{GR} = \frac{18}{e^3} \left(\frac{H^2}{2\pi\dot{\chi}}\right)^2, \quad (6.23)$$

except for the factor $18/e^3 \sim 0.896$. This difference in magnitude is due to the way we normalize the power spectrum in the uniform approximations. As shown later, the same factor also appears in the expression for the power spectrum of tensor perturbations, so that the ratio of them does not depend on this factor.

To estimate the effect from higher curvature terms on the power spectrum, let us first write the dispersion relation (6.11) in the form,

$$\omega_k^2 \equiv b_1 k^2 + b_2 \frac{k^4}{a^2 M_A^2} + b_3 \frac{k^6}{a^4 M_B^4}, \quad (6.24)$$

where [16, 17]

$$M_A \equiv |g_3|^{-1/2} M_{\text{pl}}, \quad M_B \equiv |g_8|^{-1/4} M_{\text{pl}}, \quad (6.25)$$

and

$$b_1 \equiv [1 + 2V_1 + 2\bar{A}(c_1' + c_2) - 4\pi G c_1^2 (1 - \bar{A})] / \beta_0,$$

³ Mathematically, in the limit $c_1 = 0, \beta_0 \rightarrow 1$ and $f^2/3\beta_0(\lambda - 1) \rightarrow 1$, the standard result can be restored.

$$\begin{aligned} b_2 &\equiv 2[\lambda_2 + \lambda_4 + \lambda_{23}]/\beta_0, \\ b_3 &\equiv -2[\lambda_6 + \lambda_{78}]/\beta_0, \end{aligned} \quad (6.26)$$

with

$$\begin{aligned} \lambda_2 &\equiv V_2 M_A^2, \quad \lambda_4 \equiv V_4' M_A^2, \quad \lambda_6 \equiv V_6 M_B^4, \\ \lambda_{23} &\equiv \frac{c_1^2 M_A^2}{2M_{\text{pl}}^4} (8g_2 + 3g_3), \\ \lambda_{78} &\equiv \frac{c_1^2 M_B^4}{M_{\text{pl}}^6} (8g_7 - 3g_8). \end{aligned} \quad (6.27)$$

Since g_2 and g_3 all both the coefficients of the fourth-order derivative terms, as one can see from Eq.(2.4), it is quite reasonable to assume that g_2 and g_3 are in the same order, $g_3/g_2 \simeq \mathcal{O}(1)$. Similarly, one can argue that $g_s/g_4 \simeq \mathcal{O}(1)$ for $s = 5, 6, 7, 8$, as all of these terms are the coefficients of the sixth-order derivative terms. For the sake of simplicity, we further assume $M_A \simeq M_B = M_*$. Taking $c_1 \simeq c_2 \simeq M_*$, from Eq.(1.8) we find $M_* \leq M_{\text{pl}} |c_\psi|^{1/2}$ [17]. Then, from Eq.(6.6) we obtain

$$\beta_0 \simeq f(\lambda) + \frac{1}{2} \left(\frac{M_*}{\Lambda_\omega} \right)^{4/5} \simeq \mathcal{O}(1), \quad (6.28)$$

as $f(\lambda) \simeq \mathcal{O}(1)$. To determine the scales of V_n , we assume that b_n defined in Eq.(6.26) are all of order 1, i.e.,

$$b_n \simeq \mathcal{O}(1), \quad (n = 1, 2, 3), \quad (6.29)$$

which is a reasonable assumption, considering the physical meanings of the energy scales M_A and M_B . In fact, one can define M_A and M_B so that $b_2 = b_3 = 1$ precisely, as originally defined in [17]. To have $b_1 = 1$, one can properly choose V_1 . On the other hand, since \bar{A} is undetermined, and for the sake of simplicity, we further set $\bar{A} = 0$.

With all the above assumptions, we find that the function $g(k, \eta)$ now reads,

$$\begin{aligned} g(k, \eta) &= \frac{k^2}{y^2} \left[\frac{9}{4} - y^2 (1 + \epsilon_{\text{HL}} y^2 + \epsilon_{\text{HL}}^2 y^4) \right], \\ \epsilon_{\text{HL}} &\equiv \frac{H^2}{M_*^2}. \end{aligned} \quad (6.30)$$

Thus, depending on the energy scale H when inflation occurs, one can have different turning point y_0 . In the following we consider only two limits, $\epsilon_{\text{HL}} \ll 1$ and $\epsilon_{\text{HL}} \gg 1$. In addition, in writing Eq.(6.30) we have set $b_n = 1 = \beta_0$ precisely. General expressions without setting $b_n = 1$ can be found in Appendix D.

1. $\epsilon_{\text{HL}} \ll 1$

When $\epsilon_{\text{HL}} \ll 1$, to its second order, we find that

$$y_0^2 \simeq \frac{9}{4} \left(1 - \frac{9}{4} \epsilon_{\text{HL}} + \frac{81}{16} \epsilon_{\text{HL}}^2 \right), \quad (6.31)$$

for which the power spectrum is given by,

$$P_{\mathcal{R}}(k) \simeq P_{\mathcal{R}}^{\text{GR}} \left(1 - \frac{9}{4} \epsilon_{\text{HL}} + \frac{729}{128} \epsilon_{\text{HL}}^2 \right). \quad (6.32)$$

It is interesting to note that the condition $\epsilon_{\text{HL}} \ll 1$ is equivalent to

$$V(\bar{\chi}) \ll \frac{3}{2} (3\lambda - 1) \left(\frac{\Lambda_\omega}{M_{\text{pl}}} \right)^2 M_{\text{pl}}^4, \quad (\epsilon_{\text{HL}} \ll 1). \quad (6.33)$$

2. $\epsilon_{\text{HL}} \gg 1$

When $\epsilon_{\text{HL}} \gg 1$, to find the turning point y_0 , we first write $g(k, \eta)$ given by Eq.(6.30) in the form,

$$g(k, \eta) = \epsilon_{\text{HL}}^2 \frac{k^2}{y^2} \left[\frac{9}{4} \eta_{\text{HL}}^2 - y^2 (\eta_{\text{HL}}^2 + \eta_{\text{HL}} y^2 + y^4) \right], \quad (6.34)$$

where $\eta_{\text{HL}} \equiv 1/\epsilon_{\text{HL}} \ll 1$. Then we find the perturbative solution

$$y_0^2 \simeq \left(\frac{3\eta_{\text{HL}}}{2} \right)^{2/3} \left[1 - \frac{1}{3} \left(\frac{4\eta_{\text{HL}}}{9} \right)^{1/3} - \frac{2}{9} \left(\frac{4\eta_{\text{HL}}}{9} \right)^{2/3} \right], \quad (6.35)$$

for which the power spectrum takes the form,

$$P_{\mathcal{R}}(k) \simeq P_{\mathcal{R}}^{\text{GR}}(k) \frac{4\eta_{\text{HL}} \sqrt{e}}{9} \left[1 - \frac{1}{2} \left(\frac{4\eta_{\text{HL}}}{9} \right)^{1/3} \right]. \quad (6.36)$$

Thus, if the inflation happened way above the scale M_* , the spectrum will be suppressed by the factor M_*^2/H^2 , comparing with that of GR.

B. Power Spectrum and Index of Tensor Perturbations

The tensor perturbations can be written in the form [23, 36]

$$\delta g_{ij} = a^2 (\delta_{ij} + h_{ij}), \quad (6.37)$$

where h_{ij} is traceless and transverse, i.e., $h^i_i = 0 = \partial^j h_{ij}$. For a single scalar inflaton, the anisotropic stress is zero, so the tensor perturbations are source-free. In the ADM formalism, with the results of constraint equations derived in Section II, it can be shown that the second order action is given by

$$\begin{aligned} S^{(2)} &= \frac{1}{2} \int d\eta d^3x \frac{\zeta^2 a^2}{2} \left\{ (\partial_\eta h_{ij})^2 - (1 - \bar{A}) (\partial_c h_{ij})^2 \right. \\ &\quad \left. - \frac{g_3}{\zeta^2 a^2} (\partial^2 h_{ij})^2 - \frac{g_8}{\zeta^4 a^4} (\partial_c \partial^2 h_{ij})^2 \right\}. \end{aligned} \quad (6.38)$$

Defining the following expansion in the momentum space [25],

$$h_{ij} = \int \frac{d^3k}{(2\pi)^3} \sum_{s=+, \times} \epsilon_{ij}^s(k) h_{\mathbf{k}}^s(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (6.39)$$

where $\epsilon_{ii} = k^i \epsilon_{ij} = 0$ and $\epsilon_{ij}^s(k) \epsilon_{ij}^{s'}(k) = 2\delta_{ss'}$, the above action becomes

$$S^{(2)} = \sum_{s=+, \times} \int d\eta d^3k \frac{a^2}{2\zeta^2} \left\{ (h_{\mathbf{k}}^{s'})^2 - (1 - \bar{A})k^2 (h_{\mathbf{k}}^s)^2 - \frac{g_3 k^4}{\zeta^2 a^2} (h_{\mathbf{k}}^s)^2 - \frac{g_8 k^6}{\zeta^4 a^4} (h_{\mathbf{k}}^s)^2 \right\}. \quad (6.40)$$

To make the action canonically normalized, we introduce $v_{\mathbf{k}}^s$ by

$$v_{\mathbf{k}}^s \equiv a\zeta h_{\mathbf{k}}^s. \quad (6.41)$$

Then, the action (6.40) becomes

$$S_{(2)} = \frac{1}{2} \sum_{s=+, \times} \int d\eta d^3k \left[(v_{\mathbf{k}}^{s'})^2 - (\omega_k^2 + m_{\text{eff}}^2) (v_{\mathbf{k}}^s)^2 \right], \quad (6.42)$$

but now with

$$\begin{aligned} \omega_k^2 &= (1 - \bar{A})k^2 + \frac{g_3 k^4}{\zeta^2 a^2} + \frac{g_8 k^6}{\zeta^4 a^4}, \\ m_{\text{eff}}^2 &= -\frac{a''}{a}. \end{aligned} \quad (6.43)$$

One can see that each spin state of the tensor perturbation acts like a scalar. After the quantization procedure prescribed in Appendix C, the classical equation of motion for the mode functions again read

$$v_k'' + (\omega_k^2 + m_{\text{eff}}^2) v_k = 0, \quad (6.44)$$

where in writing the above equation, we had dropped the super indices “s”, and ω_k^2 and m_{eff}^2 are now defined by Eq.(6.43). From the above, we can directly read off $g(k, \eta)$ for tensor perturbations,

$$\begin{aligned} g(k, \eta) &= \frac{k^2}{y^2} \left\{ \frac{9}{4} (1 + 2\epsilon_V) - \left[(1 - \bar{A})y^2 \right. \right. \\ &\quad \left. \left. + \frac{g_3 H^2}{\zeta^2} (1 - 2\epsilon_V) y^4 + \frac{g_8 H^4}{\zeta^4} (1 - 4\epsilon_V) y^6 \right] \right\}, \\ y &\equiv k\eta. \end{aligned} \quad (6.45)$$

Thus, its turning point $y_0^2 \equiv (k\bar{\eta})^2$ satisfies the cubic equation

$$\begin{aligned} \frac{9}{4} (1 + 2\epsilon_V) &= (1 - \bar{A})y_0^2 + \frac{g_3 H^2}{\zeta^2} (1 - 2\epsilon_V) y_0^4 \\ &\quad + \frac{g_8 H^4}{\zeta^4} (1 - 4\epsilon_V) y_0^6. \end{aligned} \quad (6.46)$$

Then the dimensionless spectrum and index for the tensor perturbations can be defined as [25],

$$P_T(k)|_{k\eta \rightarrow 0^-} \equiv 4 \times \frac{k^3}{2\pi^2} \frac{|v_k|^2}{\zeta^2 a^2} \Big|_{k\eta \rightarrow 0^-}, \quad (6.47)$$

$$n_T \equiv \frac{d \ln P_T^2}{d \ln k} \Big|_{k\eta \rightarrow 0^-}. \quad (6.48)$$

Here the factor of 4 accounts for the two spin states.

Again, assuming that the gauge field \bar{A} is constant during inflation,

$$\begin{aligned} P_T(k) &= \frac{16H^2 |y_0|^3}{3\pi^2 e^3 \zeta^2} \\ &\quad \times \exp \left[\frac{2H^2 y_0^4}{9\zeta^2} \left(g_3 + g_8 y_0^2 \frac{H^2}{\zeta^2} \right) \right], \end{aligned} \quad (6.49)$$

$$n_T = -2\epsilon_V. \quad (6.50)$$

In the relativistic limit, Eq.(6.49) yields the well-known results obtained in GR [25]

$$P_T^{\text{GR}}(k) = \frac{18}{e^3} \frac{2H^2}{\pi^2 M_{\text{pl}}^2}. \quad (6.51)$$

Because of the normalization of the power spectrum in the uniform approximation, a difference of a factor $18/e^3$ also appears in the tensor perturbations.

To study the effect of high order curvature terms, following what we did for the scalar perturbations, we consider the two cases $\epsilon_{\text{HL}} \ll 1$ and $\epsilon_{\text{HL}} \gg 1$, separately.

1. $\epsilon_{\text{HL}} \ll 1$

In this case, the power spectrum (6.49) takes the form

$$P_T(k) \simeq P_T^{\text{GR}}(k) \left(1 - \frac{9}{2} \epsilon_{\text{HL}} + \frac{729}{32} \epsilon_{\text{HL}}^2 \right). \quad (6.52)$$

Then, from Eqs.(6.32) and (6.52), we find that the scalar-tensor ratio is given by

$$r \equiv \frac{P_T(k)}{P_{\mathcal{R}}(k)} \simeq 16\epsilon_V \left(1 - \frac{9}{4} \epsilon_{\text{HL}} + \frac{2187}{128} \epsilon_{\text{HL}}^2 \right). \quad (6.53)$$

For the general case, see Eq.(D.2).

2. $\epsilon_{\text{HL}} \gg 1$

When $\epsilon_{\text{HL}} \gg 1$, from Eq.(6.45) we find that y_0^2 is given by,

$$\begin{aligned} y_0^2 \simeq & \left(\frac{3\eta_{\text{HL}}}{4} \right)^{2/3} \left[1 - \frac{1}{6} \left(\frac{16\eta_{\text{HL}}}{9} \right)^{1/3} \right. \\ & \left. - \frac{1}{18} \left(\frac{16\eta_{\text{HL}}}{9} \right)^{2/3} \right], \end{aligned} \quad (6.54)$$

and the power spectrum takes the form,

$$P_T(k) \simeq P_T^{\text{GR}}(k) \frac{2e^{1/2}\eta_{\text{HL}}}{9} \left[1 - \frac{1}{4} \left(\frac{16\eta_{\text{HL}}}{9} \right)^{1/3} \right]. \quad (6.55)$$

Then, the combination of it with Eq.(6.36) yields

$$r \simeq 8\epsilon_V \left[1 + \frac{2-2^{2/3}}{4} \left(\frac{4\eta_{\text{HL}}}{9} \right)^{1/3} \right]. \quad (6.56)$$

For arbitrary b_n , see Eq.(D.4).

VII. CONCLUSIONS

In this paper, we have studied inflation driven by a single scalar field in the HMT setup [13] with the projectability condition and an arbitrary coupling constant λ [15]. Because of the particular coupling of matter fields (3.12), in Sec. III.A we have been able to show that the FRW universe is necessarily flat for (multi-) scalar, vector and fermionic fields. It is quite reasonable to argue that this should be true for all the viable cosmological models⁴. Therefore, the HMT setup provides a built-in mechanism to solve the flatness problem. However, to solve the horizon problem inflation may or may not be needed [29], although to solve other problems, such as monopole and domain walls [25], inflation with the slow-roll conditions seems still required.

After first developing the general formulas of linear scalar perturbations without specifying a particular gauge and matter fields in Sec. III.B, we have investigated several possible gauge choices in Sec. III.C, and found that, unlike the case without the U(1) symmetry [23], now various gauge choices become possible, including the “generalized” longitudinal gauge, synchronous gauge, and quasilongitudinal gauge.

Applied the general formulas to a single scalar field, in Sec. IV we have first shown that the flat FRW universe has the same dynamics as that given in GR. As a result, all the results obtained in GR are also applicable here in the HMT setup, as far as only the background is concerned, including the slow roll conditions. Then, we have found that in the super-horizon regions, the perturbations become adiabatic, and the comoving curvature perturbation is constant, though for a different reason from that in GR.

In Sec. V, we have shown that a master equation [cf. Eq.(5.2)] for the scalar perturbations exists, in contrast to the case without the U(1) symmetry [23]. In addition, we have also shown explicitly that in the sub-horizon

regions, the metric and scalar field are tightly coupled and have the same oscillating frequencies.

We have also calculated the power spectra and spectrum indices of both the scalar and tensor perturbations in the slow-roll approximations (Sec. VI), by using the uniform approximations [26], and expressed them explicitly in terms of the slow roll parameters and the coupling constants of high order curvature terms. We’ve found that, with some reasonable conditions on the coupling coefficients $c_{1,2}$ and V_i [cf. the discussions presented after Eq.(6.23) and Eqs.(6.28) and (6.29)], the spectrum index of the tensor perturbations is the same as the value given in GR, whereas the index of the scalar perturbation is a function of λ and can be different from the standard GR value. The power spectra are in general different from those of GR. For more general cases, the power spectrum $P_{\mathcal{R}}$ and the ratio r are given in Appendix D. We have also found that inflation in the HMT setup produces all the observational features of the universe [40]. Therefore, as far as slow-roll inflation is concerned, the HL theory are consistent with observations.

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Appendix A: Field Equations

For the action (2.1), the Hamiltonian and momentum constraints are given, respectively, by,

$$\int d^3x \sqrt{g} \left[\mathcal{L}_K + \mathcal{L}_V - \varphi \mathcal{G}^{ij} \nabla_i \nabla_j \varphi - (1 - \lambda) (\Delta \varphi)^2 \right] = 8\pi G \int d^3x \sqrt{g} J^t, \quad (A.1)$$

$$\nabla^j \left[\pi_{ij} - \varphi \mathcal{G}_{ij} - (1 - \lambda) g_{ij} \nabla^2 \varphi \right] = 8\pi G J_i, \quad (A.2)$$

where

$$J^t \equiv 2 \frac{\delta(N\mathcal{L}_M)}{\delta N}, \quad J_i \equiv -N \frac{\delta \mathcal{L}_M}{\delta N^i},$$

$$\pi^{ij} \equiv \frac{\delta(N\mathcal{L}_K)}{\delta \dot{g}_{ij}} = -K^{ij} + \lambda K g^{ij}. \quad (A.3)$$

Variation of the action (2.1) with respect to φ and A yield,

$$\mathcal{G}^{ij} \left(K_{ij} + \nabla_i \nabla_j \varphi \right) + (1 - \lambda) \Delta \left(K + \Delta \varphi \right) = 8\pi G J_\varphi, \quad (A.4)$$

$$R - 2\Lambda_g = 8\pi G J_A, \quad (A.5)$$

where

$$J_\varphi \equiv -\frac{\delta \mathcal{L}_M}{\delta \varphi}, \quad J_A \equiv 2 \frac{\delta(N\mathcal{L}_M)}{\delta A}. \quad (A.6)$$

⁴ It should be noted that this conclusion is based on the coupling of matter fields to the gauge field A and Newtonian prepotential φ , proposed in [15].

On the other hand, the dynamical equations now read⁵,

$$\begin{aligned}
& \frac{1}{N\sqrt{g}} \left\{ \sqrt{g} \left[\pi^{ij} - \varphi \mathcal{G}^{ij} - (1-\lambda) g^{ij} \Delta \varphi \right] \right\}_{,t} \\
&= -2 (K^2)^{ij} + 2\lambda K K^{ij} - \frac{2}{N} \pi^{k(i} \nabla_k N^{j)} \\
&+ \nabla_k \left[\frac{N^k}{N} \pi^{ij} - (1-\lambda) F_\varphi^k g^{ij} \right] \\
&- 2(1-\lambda) \left[(K + \Delta \varphi) \nabla^i \nabla^j \varphi + K^{ij} \Delta \varphi \right] \\
&+ 2(1-\lambda) \nabla^{(i} \left[\nabla^{j)} \varphi (K + \Delta \varphi) \right] \\
&+ \frac{1-\lambda}{N} \Delta \varphi \nabla^{(i} N^{j)} \\
&+ \frac{1}{2} (\mathcal{L}_K + \mathcal{L}_\varphi + \mathcal{L}_A + \mathcal{L}_\lambda) g^{ij} \\
&+ F^{ij} + F_\varphi^{ij} + F_A^{ij} + 8\pi G \tau^{ij}, \tag{A.7}
\end{aligned}$$

where $(K^2)^{ij} \equiv K^{il} K_l^j$, $f_{(ij)} \equiv (f_{ij} + f_{ji})/2$, and

$$\begin{aligned}
F^{ij} &\equiv \frac{1}{\sqrt{g}} \frac{\delta(-\sqrt{g} \mathcal{L}_V)}{\delta g_{ij}} = \sum_{s=0}^8 g_s \zeta^{ns} (F_s)^{ij}, \\
F_\varphi^{ij} &= \sum_{n=1}^3 F_{(\varphi,n)}^{ij}, \\
F_\varphi^i &= (K + \nabla^2 \varphi) \nabla^i \varphi + \frac{N^i}{N} \Delta \varphi, \\
F_A^{ij} &= \frac{1}{N} \left[A R^{ij} - (\nabla^i \nabla^j - g^{ij} \Delta) A \right], \tag{A.8}
\end{aligned}$$

with $n_s = (2, 0, -2, -2, -4, -4, -4, -4, -4)$. $(F_s)_{ij}$ and $F_{(\varphi,n)}^{ij}$ are given by [14, 23],

$$\begin{aligned}
(F_0)_{ij} &= -\frac{1}{2} g_{ij}, \\
(F_1)_{ij} &= -\frac{1}{2} g_{ij} R + R_{ij}, \\
(F_2)_{ij} &= -\frac{1}{2} g_{ij} R^2 + 2R R_{ij} - 2\nabla_{(i} \nabla_{j)} R \\
&\quad + 2g_{ij} \nabla^2 R, \\
(F_3)_{ij} &= -\frac{1}{2} g_{ij} R_{mn} R^{mn} + 2R_{ik} R_j^k - 2\nabla^k \nabla_{(i} R_{j)k} \\
&\quad + \nabla^2 R_{ij} + g_{ij} \nabla_m \nabla_n R^{mn}, \\
(F_4)_{ij} &= -\frac{1}{2} g_{ij} R^3 + 3R^2 R_{ij} - 3\nabla_{(i} \nabla_{j)} R^2 \\
&\quad + 3g_{ij} \nabla^2 R^2,
\end{aligned}$$

$$\begin{aligned}
(F_5)_{ij} &= -\frac{1}{2} g_{ij} R R^{mn} R_{mn} + R_{ij} R^{mn} R_{mn} \\
&\quad + 2R R_{ki} R_j^k - \nabla_{(i} \nabla_{j)} (R^{mn} R_{mn}) \\
&\quad - 2\nabla^n \nabla_{(i} R R_{j)n} + g_{ij} \nabla^2 (R^{mn} R_{mn}) \\
&\quad + \nabla^2 (R R_{ij}) + g_{ij} \nabla_m \nabla_n (R R^{mn}), \\
(F_6)_{ij} &= -\frac{1}{2} g_{ij} R_n^m R_p^n R_m^p + 3R^{mn} R_{ni} R_{mj} \\
&\quad + \frac{3}{2} \nabla^2 (R_{in} R_j^n) + \frac{3}{2} g_{ij} \nabla_k \nabla_l (R_n^k R^{ln}) \\
&\quad - 3\nabla_k \nabla_{(i} (R_{j)n} R^{nk}), \\
(F_7)_{ij} &= -\frac{1}{2} g_{ij} (\nabla R)^2 + (\nabla_i R) (\nabla_j R) - 2R_{ij} \nabla^2 R \\
&\quad + 2\nabla_{(i} \nabla_{j)} \nabla^2 R - 2g_{ij} \nabla^4 R, \\
(F_8)_{ij} &= -\frac{1}{2} g_{ij} (\nabla_p R_{mn}) (\nabla^p R^{mn}) - \nabla^4 R_{ij} \\
&\quad + (\nabla_i R_{mn}) (\nabla_j R^{mn}) + 2(\nabla_p R_{in}) (\nabla^p R_j^n) \\
&\quad + 2\nabla^n \nabla_{(i} \nabla^2 R_{j)n} + 2\nabla_n (R_m^n \nabla_{(i} R_{j)n}^m) \\
&\quad - 2\nabla_n (R_{m(j} \nabla_{i)} R^{mn}) - 2\nabla_n (R_{m(i} \nabla^n R_{j)n}^m) \\
&\quad - g_{ij} \nabla^n \nabla^m \nabla^2 R_{mn}, \tag{A.9}
\end{aligned}$$

and

$$\begin{aligned}
F_{(\varphi,1)}^{ij} &= \frac{1}{2} \varphi \left\{ (2K + \nabla^2 \varphi) R^{ij} \right. \\
&\quad - 2(2K_k^j + \nabla^j \nabla_k \varphi) R^{ik} \\
&\quad - 2(2K_k^i + \nabla^i \nabla_k \varphi) R^{jk} \\
&\quad \left. - (2\Lambda_g - R) (2K^{ij} + \nabla^i \nabla^j \varphi) \right\}, \\
F_{(\varphi,2)}^{ij} &= \frac{1}{2} \nabla_k \left[2(\varphi \mathcal{G}^{k(i} \nabla^{j)} \varphi) - \varphi \mathcal{G}^{ij} \left(\frac{2N^k}{N} + \nabla^k \varphi \right) \right] \\
&\quad + \frac{2\varphi}{N} \mathcal{G}^{k(i} \nabla_k N^{j)}, \\
F_{(\varphi,3)}^{ij} &= \frac{1}{2} \left\{ 2\nabla_k \nabla^{(i} f_\varphi^{j)k} - \nabla^2 f_\varphi^{ij} - (\nabla_k \nabla_l f_\varphi^{kl}) g^{ij} \right\}, \tag{A.10}
\end{aligned}$$

where

$$f_\varphi^{ij} = \varphi \left\{ (2K^{ij} + \nabla^i \nabla^j \varphi) - \frac{1}{2} (2K + \nabla^2 \varphi) g^{ij} \right\}. \tag{A.11}$$

The stress 3-tensor τ^{ij} is defined as

$$\tau^{ij} = \frac{2}{\sqrt{g}} \frac{\delta(\sqrt{g} \mathcal{L}_M)}{\delta g_{ij}}. \tag{A.12}$$

The conservation laws of energy and momentum of matter fields read, respectively,

$$\int d^3x \sqrt{g} \left[\dot{g}_{kl} \tau^{kl} - \frac{1}{\sqrt{g}} (\sqrt{g} J^t)_{,t} + \frac{2N_k}{N\sqrt{g}} (\sqrt{g} J^k)_{,t} \right]$$

⁵ Note that the dynamical equations given here differ from those given in [16] because here we took N^i as the fundamental variable instead of N_i as what we did in [16]. They are both self-consistent if N^i and N_i are used consistently.

$$-2\dot{\varphi}J_\varphi - \frac{A}{N\sqrt{g}}(\sqrt{g}J_A)_{,t} = 0, \quad (\text{A.13})$$

$$\begin{aligned} \nabla^k \tau_{ik} - \frac{1}{N\sqrt{g}}(\sqrt{g}J_i)_{,t} - \frac{J^k}{N}(\nabla_k N_i - \nabla_i N_k) \\ - \frac{N_i}{N}\nabla_k J^k + J_\varphi \nabla_i \varphi - \frac{J_A}{2N}\nabla_i A = 0. \end{aligned} \quad (\text{A.14})$$

Appendix B: Scalar Fields

How matter couples with gravity in the HMT setup has not yet been worked out in the general case. In this Appendix, we shall consider the coupling of a scalar field χ with gravity by the prescription given in [15]. The case with detailed balance condition softly breaking was studied in [33].

A. Coupling of a Scalar Field

When the scalar field coupled only with N , N^i , g_{ij} , the most general action takes the form [28, 34],

$$S_\chi^{(0)}(N, N^i, g_{ij}; \chi) = \int dt d^3x N \sqrt{g} \mathcal{L}_\chi^{(0)}(N, N^i, g_{ij}; \chi), \quad (\text{B.1})$$

where

$$\begin{aligned} \mathcal{L}_\chi^{(0)} &= \frac{f(\lambda)}{2N^2}(\dot{\chi} - N^i \nabla_i \chi)^2 - \mathcal{V}, \\ \mathcal{V} &= V(\chi) + \left(\frac{1}{2} + V_1(\chi)\right)(\nabla\chi)^2 + V_2(\chi)\mathcal{P}_1^2 \\ &\quad + V_3(\chi)\mathcal{P}_1^3 + V_4(\chi)\mathcal{P}_2 + V_5(\chi)(\nabla\chi)^2\mathcal{P}_2 \\ &\quad + V_6(\chi)\mathcal{P}_1\mathcal{P}_2, \end{aligned} \quad (\text{B.2})$$

with $V(\chi)$ and $V_n(\chi)$ being arbitrary functions of χ , and

$$\mathcal{P}_n \equiv \Delta^n \chi. \quad (\text{B.3})$$

Note that in the kinetic term we added a factor $f(\lambda)$, which is an arbitrary function of λ , subjected to the requirements: (i) The scalar field must be ghost-free in all the energy scales, including the UV and IR. (ii) In the IR limit, the scalar field has a well-defined velocity, which should be equal or very closed to its relativistic value. (iii) The stability condition in the IR requires [33],

$$f(\lambda) > 0. \quad (\text{B.4})$$

To couple with the gauge field A and the Newtonian prepotential φ , we make the replacement [15],

$$S_\chi^{(0)}(N, N^i, g_{ij}; \chi) \rightarrow S_\chi(N, N^i, g_{ij}, A, \varphi; \chi), \quad (\text{B.5})$$

where

$$\begin{aligned} S_\chi(N, N^i, g_{ij}, A, \varphi; \chi) &\equiv S_\chi^A(\chi, A) \\ &\quad + S_\chi^{(0)}(N, (N^i + N\nabla^i \varphi), g_{ij}; \chi), \end{aligned} \quad (\text{B.6})$$

with

$$S_\chi^A \equiv \int dt d^3x \sqrt{g} \left[c_1(\chi) \Delta\chi + c_2(\chi) (\nabla\chi)^2 \right] (A - \mathcal{A}). \quad (\text{B.7})$$

Thus, the action can be cast in the form,

$$S_\chi = \int dt d^3x N \sqrt{g} \mathcal{L}_\chi, \quad (\text{B.8})$$

where

$$\begin{aligned} \mathcal{L}_\chi &= \mathcal{L}_\chi^{(0)} + \mathcal{L}_\chi^{(A, \varphi)}, \\ \mathcal{L}_\chi^{(A, \varphi)} &= \frac{A - \mathcal{A}}{N} \left[c_1 \Delta\chi + c_2 (\nabla\chi)^2 \right] \\ &\quad - \frac{f}{N} (\dot{\chi} - N^i \nabla_i \chi) (\nabla^k \varphi) (\nabla_k \chi) \\ &\quad + \frac{f}{2} [(\nabla^k \varphi) (\nabla_k \chi)]^2, \end{aligned} \quad (\text{B.9})$$

with $\mathcal{L}_\chi^{(0)}$ given by Eq.(B.2). Then, we find that

$$\begin{aligned} J^t &= -2 \left(\frac{f}{2N^2} (\dot{\chi} - N^k \nabla_k \chi)^2 + \mathcal{V} \right) \\ &\quad - [c_1 \Delta\chi + c_2 (\nabla\chi)^2] (\nabla\varphi)^2 \\ &\quad + f [(\nabla^k \varphi) (\nabla_k \chi)]^2, \end{aligned} \quad (\text{B.10})$$

$$\begin{aligned} J^i &= \frac{f}{N} \left[\dot{\chi} - (N^k + N\nabla^k \varphi) (\nabla_k \chi) \right] \nabla^i \chi \\ &\quad + [c_1 \Delta\chi + c_2 (\nabla\chi)^2] \nabla^i \varphi, \end{aligned} \quad (\text{B.11})$$

$$\begin{aligned} J_\varphi &= \frac{1}{N\sqrt{g}} \left\{ \sqrt{g} [c_1 \Delta\chi + c_2 (\nabla\chi)^2] \right\}_{,t} \\ &\quad - \frac{1}{N} \nabla_i \left\{ f [\dot{\chi} - (N^k + N\nabla^k \varphi) (\nabla_k \chi)] \nabla^i \chi \right. \\ &\quad \left. + [c_1 \Delta\chi + c_2 (\nabla\chi)^2] \right. \\ &\quad \left. \times (N^i + N\nabla^i \varphi) \right\}, \end{aligned} \quad (\text{B.12})$$

$$J_A = 2 [c_1 \Delta\chi + c_2 (\nabla\chi)^2], \quad (\text{B.13})$$

$$\tau_{ij} = \tau_{ij}^{(0)} + \tau_{ij}^\varphi, \quad (\text{B.14})$$

where

$$\begin{aligned} \tau_{ij}^{(0)} &= g_{ij} \left\{ \mathcal{L}_\chi^{(0)} + \nabla_k [(\mathcal{V}_{,1} + \Delta\mathcal{V}_{,2}) \nabla^k \chi + \mathcal{V}_{,2} \nabla^k \Delta\chi] \right\} \\ &\quad + (1 + 2V_1 + 2V_5 \mathcal{P}_2) (\nabla_i \chi) (\nabla_j \chi) \\ &\quad - 2(\nabla_{(i} \mathcal{V}_{,1}) (\nabla_{j)} \chi) - 2(\nabla_{(i} \Delta\mathcal{V}_{,2}) (\nabla_{j)} \chi) \\ &\quad - 2(\nabla_{(i} \mathcal{V}_{,2}) (\nabla_{j)} \Delta\chi), \\ \tau_{ij}^\varphi &= g_{ij} \left\{ \mathcal{L}_\chi^{(A, \varphi)} - \frac{1}{N} \nabla_k [c_1 (A - \mathcal{A}) \nabla^k \chi] \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{2(\mathcal{A} - A)}{N} \left[c_1 \nabla_i \nabla_j \chi + c_2 (\nabla_i \chi) (\nabla_j \chi) \right] \\
& + \left[c_1 \Delta \chi + c_2 (\nabla \chi)^2 \right] (\nabla_i \varphi) (\nabla_j \varphi) \\
& + \frac{2f}{N} \left[\dot{\chi} - (N^k + N \nabla^k \varphi) (\nabla_k \chi) \right] (\nabla_i \chi) (\nabla_j \varphi) \\
& + \frac{2}{N} \nabla_i \left[c_1 (A - \mathcal{A}) \nabla_j \chi \right], \tag{B.15}
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{V}_{,1} & \equiv \frac{\partial \mathcal{V}}{\partial \mathcal{P}_1} = 2V_2 \mathcal{P}_1 + 3V_3 \mathcal{P}_1^2 + V_6 \mathcal{P}_2, \\
\mathcal{V}_{,2} & \equiv \frac{\partial \mathcal{V}}{\partial \mathcal{P}_2} = V_4 + V_5 (\nabla \chi)^2 + V_6 \mathcal{P}_1. \tag{B.16}
\end{aligned}$$

On the other hand, the variation of the action (B.8) with respect to χ yields the following generalized Klein-Gordon equation,

$$\begin{aligned}
& \frac{f}{N\sqrt{g}} \left\{ \frac{\sqrt{g}}{N} \left[\dot{\chi} - (N^k + N \nabla^k \varphi) \nabla_k \chi \right] \right\}_{,t} \\
& = \frac{f}{N^2} \nabla_i \left\{ \left[\dot{\chi} - (N^k + N \nabla^k \varphi) \nabla_k \chi \right] (N^i + N \nabla^i \varphi) \right\} \\
& + \frac{2g^{ij}}{N} \nabla_i \left\{ \nabla_j \left[(A - \mathcal{A}) c_1 \right] - (A - \mathcal{A}) c_2 \nabla_j \chi \right\} \\
& + \frac{A - \mathcal{A}}{N} \left[c_1' \Delta \chi + c_2' (\nabla \chi)^2 \right] \\
& + \nabla^i \left[(1 + 2V_1 + 2V_5 \mathcal{P}_2) \nabla_i \chi \right] \\
& - \mathcal{V}_{,\chi} - \Delta (\mathcal{V}_{,1}) - \Delta^2 (\mathcal{V}_{,2}), \tag{B.17}
\end{aligned}$$

where $c_1' \equiv dc_1(\chi)/d\chi$, and

$$\begin{aligned}
\mathcal{V}_{,\chi} & \equiv \frac{\partial \mathcal{V}}{\partial \chi} = V' + V_1' (\nabla \chi)^2 + V_2' \mathcal{P}_1^2 + V_3' \mathcal{P}_1^3 \\
& + V_4' \mathcal{P}_2 + V_5' (\nabla \chi)^2 \mathcal{P}_2 + V_6' \mathcal{P}_1 \mathcal{P}_2. \tag{B.18}
\end{aligned}$$

B. Linear Perturbations under the Newtonian Quasilon longitudinal Gauge

Under the gauge (5.1), Eqs.(4.8) - (4.15) can be cast in the forms,

$$\begin{aligned}
& \int d^3 x \left\{ \partial^2 \psi - \frac{1}{2} (3\lambda - 1) \mathcal{H} [3\psi' + \partial^2 B] \right\} \\
& = 4\pi G \int d^3 x \left\{ f \bar{\chi}' \delta \chi' + \left(a^2 V' + \frac{V_4}{a^2} \partial^4 \right) \delta \chi \right\}, \tag{B.19} \\
& \int d^3 x a^2 \bar{\chi}' \left\{ f \delta \chi'' + 2\mathcal{H} f \delta \chi' + a^2 V'' \delta \chi - 3f \bar{\chi}' \psi' \right. \\
& \quad \left. - \frac{\bar{A} (ac_1 \delta \chi)'}{\bar{\chi}' a} \right\}
\end{aligned}$$

$$= - \int d^3 x \partial^4 \left\{ V_4 \delta \chi' + (V_4' \bar{\chi}' - V_4 \mathcal{H}) \delta \chi \right\}, \tag{B.20}$$

$$(3\lambda - 1) \psi' - (1 - \lambda) \partial^2 B = 8\pi G f \bar{\chi}' \delta \chi, \tag{B.21}$$

$$\begin{aligned}
& 2\mathcal{H} \psi + (1 - \lambda) (3\psi' + \partial^2 B) \\
& = 8\pi G \left[(c_1' \bar{\chi}' + c_1 \mathcal{H} - f \bar{\chi}') \delta \chi + c_1 \delta \chi' \right], \tag{B.22}
\end{aligned}$$

$$\psi = 4\pi G c_1 \delta \chi, \tag{B.23}$$

$$\begin{aligned}
& \psi'' + 2\mathcal{H} \psi' + \frac{1}{3} \partial^2 (B' + 2\mathcal{H} B) \\
& - \frac{2}{3(3\lambda - 1)} \left(1 + \frac{\alpha_1}{a^2} \partial^2 + \frac{\alpha_2}{a^4} \partial^4 \right) \partial^2 \psi \\
& + \frac{2}{3(3\lambda - 1)a} \partial^2 (\hat{A} \psi - \delta A) \\
& = \frac{8\pi G}{3\lambda - 1} (f \bar{\chi}' \delta \chi' - a^2 V' \delta \chi), \tag{B.24}
\end{aligned}$$

$$\begin{aligned}
& \psi - (B' + 2\mathcal{H} B) + \frac{1}{a^2} (\alpha_1 + \frac{\alpha_2}{a^2} \partial^2) \partial^2 \psi \\
& - \frac{1}{a} (\hat{A} \psi - \delta A) = 0, \tag{B.25}
\end{aligned}$$

$$\begin{aligned}
& f \left\{ \delta \chi'' + 2\mathcal{H} \delta \chi' - \bar{\chi}' [3\psi' + \partial^2 B] \right\} + a^2 V'' \delta \chi \\
& = 2 \left(\frac{1}{2} + V_1 - \frac{V_2 + V_4'}{a^2} \partial^2 - \frac{V_6}{a^4} \partial^4 \right) \partial^2 \delta \chi \\
& + \frac{1}{a} \partial^2 [2\hat{A} (c_1' - c_2) \delta \chi + c_1 \delta A]. \tag{B.26}
\end{aligned}$$

Recall $\hat{A} = a\bar{A}$. It can be shown that Eqs.(B.22) and (B.24) are not independent, and can be obtained from the others. Therefore, in the present case there are four independent differential equations, (B.21), (B.23), (B.25), and (B.26), for the four unknowns, ψ , B , δA and $\delta \chi$.

Appendix C: Quantization of scalar perturbations

This part summarizes the discussions given in [25].⁶ To quantize the scalar field, suppose we have the normalized action of second order

$$S^{(2)} = \frac{1}{2} \int d\eta d^3 x \left[v'^2 - \beta(\eta, \partial^n) v^2 \right], \tag{C.1}$$

where $\beta(\eta, \partial^n)$ is in general time-dependent explicitly and $\partial^n = \partial^2, \partial^4, \partial^6, \dots$. Now promote the field v and its conjugate momentum to operators,

$$v \rightarrow \hat{v}(\eta, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} \left[v_k(\eta) \hat{a}_{\mathbf{k}} e^{i\mathbf{k}\mathbf{x}} + v_k^*(\eta) \hat{a}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\mathbf{x}} \right]. \tag{C.2}$$

⁶ Note that here we did not consider any modifications of the commutation relations [39].

If we define the Fourier image of $v(\eta, \mathbf{x})$ as

$$v(\tau, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} v_{\mathbf{k}}(\eta) e^{i\mathbf{k}\mathbf{x}}, \quad (\text{C.3})$$

this is equivalent to say that

$$v_{\mathbf{k}}(\eta) \rightarrow \hat{v}_{\mathbf{k}}(\eta) = v_{\mathbf{k}}(\eta) \hat{a}_{\mathbf{k}} + v_{\mathbf{k}}^*(\eta) \hat{a}_{\mathbf{k}}^\dagger. \quad (\text{C.4})$$

$v_{\mathbf{k}}(\tau)$ are called the *mode functions*. They satisfy the second order classical equation of motion (EoM)

$$v_{\mathbf{k}}'' + \beta(\eta, k^{2n}) v_{\mathbf{k}} = 0. \quad (\text{C.5})$$

The canonical commutation relation between quantum field $\hat{v}_{\mathbf{k}}$ and its conjugate momentum $\hat{v}_{\mathbf{k}}'$ is,

$$\langle 0 | [\hat{v}_{\mathbf{k}}, \hat{v}_{\mathbf{k}'}'] | 0 \rangle = i\hbar. \quad (\text{C.6})$$

If we want to have

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}'), \quad (\text{C.7})$$

the norm (Wronskian) has to be

$$v_{\mathbf{k}}^* v_{\mathbf{k}}' - v_{\mathbf{k}}'^* v_{\mathbf{k}} = -i\hbar. \quad (\text{C.8})$$

Besides the normalization condition, we need another boundary condition to determine the mode functions completely. Usually this is obtained by requiring that the vacuum state to be the ground state of the Hamiltonian back in the far past when the mode is deep inside horizon

$$\hat{H} |0\rangle = E_0 |0\rangle, \quad (\text{C.9})$$

where the vacuum is defined as $\hat{a}_{\mathbf{k}} |0\rangle = 0$. Since we have

$$\hat{H} = \frac{1}{2} (\hat{v}_{\mathbf{k}}'^2 + \beta \hat{v}_{\mathbf{k}}^2), \quad (\text{C.10})$$

this requires $v_{\mathbf{k}}' = \pm i\sqrt{\beta} v_{\mathbf{k}}$ if $|0\rangle$ is the ground state. Thus

$$v_{\mathbf{k}} = C e^{-i \int d\eta \sqrt{\beta}}, \quad (\text{C.11})$$

where the positive frequency branch is selected to ensure the positivity of normalization, and C will be determined by the normalization condition (C.8).

Appendix D: General expressions for power spectra

Here we give the more general expressions of spectra without setting $b_1 = b_2 = b_3 = 1 = \beta_0$. $M_A = M_B = M_*$ (and thus by (6.25) $g_3^2 = g_8$) and $\dot{A} = 0$ is still assumed. In the limiting case when $\epsilon_{\text{HL}} \equiv (H/M_*)^2 \ll 1$, we find

$$P_{\mathcal{R}}(k) \simeq P_{\mathcal{R}}^{\text{GR}} \frac{1}{(b_1)^{2/3} \beta_0} \left(1 + \frac{c_1 \dot{\chi}}{2M_{\text{pl}}^2 H} \right)^2 \times \left[1 - \frac{9b_2}{4b_1^2} \epsilon_{\text{HL}} + \frac{81(17b_2^2 - 8b_1 b_3)}{128b_1^4} \epsilon_{\text{HL}}^2 \right], \quad (\text{D.1})$$

$$r \simeq 16\epsilon_V (b_1)^{2/3} \beta_0 \left(1 + \frac{c_1 \dot{\chi}}{2M_{\text{pl}}^2 H} \right)^{-2} \times \left[1 + \frac{9(b_2 - 2b_1^2)}{4b_1^2} \epsilon_{\text{HL}} + \frac{81(36b_1^4 - 17b_2^2 + 8b_1 b_3)}{128b_1^4} \epsilon_{\text{HL}}^2 \right]. \quad (\text{D.2})$$

In the limit $\epsilon_{\text{HL}} \gg 1$, we obtain

$$P_{\mathcal{R}}(k) \simeq P_{\mathcal{R}}^{\text{GR}} \frac{4e^{\frac{1}{2}} \eta_{\text{HL}}}{9\sqrt{b_3} \beta_0} \left(1 + \frac{c_1 \dot{\chi}}{2M_{\text{pl}}^2 H} \right)^2 \times \left[1 - \frac{b_2}{2b_3} \left(\frac{4b_3}{9} \eta_{\text{HL}} \right)^{\frac{1}{3}} \right], \quad (\text{D.3})$$

$$r \simeq 16\epsilon_V \frac{\sqrt{b_3} \beta_0}{2} \left(1 + \frac{c_1 \dot{\chi}}{2M_{\text{pl}}^2 H} \right)^{-2} \times \left\{ 1 - \left[\frac{1}{4} - \frac{b_2}{2b_3} \left(\frac{b_3}{4} \right)^{1/3} \right] \left(\frac{16}{9} \eta_{\text{HL}} \right)^{\frac{1}{3}} \right\}. \quad (\text{D.4})$$

Clearly, the magnitude of the ratio r are dependent on the values of b_1 and b_3 .

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